Phys 535 **Lab Session 10**

Question 1 *Sound waves*

The code for this problem can be checked out via CVS as `wdobler/Phys535/idl/sound`.

(a) Solve the equations for sound waves in 1-d:

$$\begin{aligned} \partial_t \ln \varrho &= -v\,\partial_x \ln \varrho - \partial_x v \;, \\ \partial_t v &= -v\partial_x v - c_{\mathrm{s}}^2 \partial_x \ln \varrho + \frac{4}{3}\nu\partial_x^2 v \end{aligned}$$

on the (non-periodic) interval $0 \le x < 1$ for a Gasussian profile as initial condition.

(b) Adapt the initial condition such that the bump moves only to the right (and stick with that for the following questions).

(c) Increase the amplitude to `ampl` $= 0.3$. What changes? For how long can you run the simulation?

(d) How much viscosity do you need to stabilize the run for an amplitude `ampl` $= 2$?

(e) What do the boundary conditions represent physically? Modify the right boundary condition to represent the open end of a pipe (e.g. in a wind instrument).

(f) Why can we use periodic derivative operators (they use IDL's cyclic `shift` function) for this non-periodic problem?

Question 2 *Schrödinger equation*

Solve the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \;,$$

where

$$H = -\frac{\hbar^2}{2\,m}\frac{\partial^2}{\partial x^2} + U(x) \;.$$

for periodic boundary conditions on the interval $0 \le x < 2\pi$. Use units where $\hbar = 1$ and $m = 1$.

The initial condition is a wave packet

$$\psi = \frac{1}{(\pi\,w^2)^{1/4}} e^{-\frac{1}{2}\left(\frac{x - x_{\mathrm{peak}}}{w}\right)^2 + i\frac{p_0 x}{\hbar}} \;,$$

where $x_{\mathrm{peak}} = \pi$, $w = 0.3$, $p_0 = 15$.

The potential is

$$U(x) = 150\,\frac{1 + \tanh\frac{\sin(x-4) - 0.95}{0.002}}{2}$$

Use at least 200 points. Monitor the total probability $\int |\psi|^2\,dx$ and increase the number of points if it varies noticeably.

Work *collectively* on this problem and check in your contributions under `common/schroedinger`.

(a) Plot $|\psi|$, $|\psi|^2$, $\Re\psi$, and $\Im\psi$

(b) Setting the potential to zero, how does the motion of the wave packet change when you change p_0? What is the physical meaning of p_0?

(c) What changes if you use a narrower wave packet?

(d) [Revert to the original width and potential] Vary the height of the potential barrier and explain the resulting changes.