Question 1 *Simple heat conduction with high-order method*

Some code for this problem can be checked out via CVS as `wdobler/Phys535/idl/heatcond`.

Solve the heat conduction equation

$$\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial x^2}$$

on the periodic interval $0 \le x < 2\pi$, where thermal diffusivity $\chi = 1$ is constant. The initial condition is

$$T(x, 0) = \sum_{n=1}^{8} (-1)^{n-1} \frac{\sin nx}{n}$$

Plot the temperature profile and the heat flux density $F = -\partial T/\partial x$ for about 5 interesting times.

Find the Courant time step (border between stable and unstable).

Hint: You can use IDL's `deriv()` or our `xder()` to calculate the first derivative.

Question 2 *Advection*

The code for this problem can be checked out via CVS as `wdobler/Phys535/idl/advect`.

Solve the advection equation

$$\frac{\partial f}{\partial t} = -u \frac{\partial f}{\partial x}$$

on the periodic interval $0 \le x < 1$, where the advection velocity $u = 1$ is constant. The initial condition is

$$f(x, 0) = \tanh(7 \sin x) \ .$$

(a) Run for 10 time units and compare the final profile to the original one.

(b) Try to understand what the different files do. What is new compared to the systems of ordinary differential equations we have solved earlier?

(c) Find the Courant time step.

(d) Set numerical diffusivity `visc` to zero; what happens? How does the Euler time-stepping method fare in the absence of diffusivity?

(e) Use different spatial derivative schemes and compare. Does the spectral scheme really give the exact result (what happens if you decrease the time step?)?

(f) Switch to 6th-order hyperdiffusivity and find an acceptable value of `visc`. How does the energy decrease compare to 2nd-order normal diffusivity?

(g) If you do not like the deformation of the signal, you can increase the number of points.