Question 1 *One-dimensional Ising model*

Following the outline given in the lecture, program the one-dimensional Ising model

$$E = -\varepsilon \sum_{i=0}^{N-1} s_i(s_{i-1}+s_{i+1})$$

for N spins. Assume periodic boundary conditions, i.e. $s_N = s_0$, $s_{-1} = s_{N-1}$.

(a) Using $N = 20$ points, plot energy per spin E/N and magnetization per spin $M/N = \sum s_i/N$ as functions of temperature T.

(b) Plot the specific heat c_v and the magnetic susceptibility χ as functions of temperature T.

(c) If you double the number of points, do you get markedly different results?

(d) What are the options to vectorize the calculation?

Hints:

- Do not forget to discard a number of early values while the system tries to find its equilibrium. The lower the temperature, the longer equilibration will take.
- The IDL function `shift(`*vector*`,`*n*`)` takes a vector and performs a cyclic shift of its elements by n.

Question 2 *Quantum Monte Carlo*

Using the trial wave function

$$\Phi(x) = Axe^{-kx} , \tag{1}$$

find a Monte Carlo approximation E to the energy E_0 of the ground state for a particle in the potential well

$$U(x) = \begin{cases} \infty , & x < 0 , \\ x , & x \geq 0 . \end{cases} \tag{2}$$

(a) Find $E(k)$ for a set of values of the parameter k, at least $\{0.5, 1, 1.5, 2\}$.

(b) If you choose a normal distribution for the displacement instead of a uniform one, do the energies change? Can you explain?

Question 3 *Simulated annealing*

Use *simulated annealing* to find an approximation to the global energy minimum for the one-dimensional Ising model.

Hint: Use your program(s) from Question 1.