Question 1 *Monte Carlo integration*

(a) Use Monte Carlo integration to calculate the volume of the unit sphere.

(b) Now generalize your code to calculate the d-hypervolume of the d-dimensional unit hypersphere for $d = 1, 2, 3, \ldots, 10$.

Question 2 *Quasi-random numbers*

Note: Quasi-ransom numbers are generated by the function `sobseq()`. To learn how to call that function, you can do `dummy=sobseq(/HELP)`.

(a) Use Quasi-Monte Carlo integration to calculate the area of the unit circle. Compare the accuracy to Monte Carlo integration.

(b) Plot the first 128 quasi-random points in two dimensions (don't forget to reset the generator). Now overplot points number 129 to 512 in another colour, and then points513 to 1024 in yet another colour.

Repeat this with uniformly distributed random numbers and compare.

(c) Verify the isotropy of quasi-random numbers on $[-1, 1] \times [-1, 1]$ by counting the numbers in annular segments

$$0.8 < r < 1 \,, \qquad \varphi_i < \varphi \le \varphi_i + \delta\varphi$$

where you need to choose the number of points and the size $\delta\varphi$ of the angular bins appropriately.

Repeat this with uniformly distributed random numbers.

Now use the same diagnostics, but choose (quasi) random points from $[-1, 1] \times [-4, 4]$; again, do this for quasi random numbers and for uniformly distributed random numbers. Conclusion?

Question 3 *Random walk in 1 dimension*

A particle starts at $x = 0$ for $t = 0$ and then moves by Δ during the time interval δt. The values of Δ for different time steps are stochastically independent.

We expect the probability density function of the x-position X to spread according to the diffusion equation

$$\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial t^2} \,,$$

which has the Green's function

$$G(x-x', t-t') = \frac{1}{\sqrt{4\pi D(t-t')}}\, e^{-\frac{(x-x')^2}{4D(t-t')}} \tag{1}$$

(a) Assume $\Delta = \pm 1$, where the sign is randomly chosen for each time step. Plot the probability density function after $N_t = 1000$ steps and try to fit a Gaussian (1) to it. What is the diffusion constant D? How can you test whether the broadening of the PDF is really described by Eq. (1)?

(b) Repeat part (a) for $\Delta \sim \mathcal{U}(-0.5, 0.5)$.

(c) Returning to $\Delta = \pm 1$, try to obtain the PDF of the *return time*, i.e. the time it takes for one particle to return to the origin for the first time. Will the average return time be finite?