Question 1 *Multi-dimensional root finding in IDL*

(a) Numerically, find the position of the maxima of

$$f(x) = \frac{x^3}{e^x - 1} \tag{1}$$

and

$$g(x) = \frac{1/x^5}{e^{1/x} - 1} \; . \tag{2}$$

Use IDL's `broyden()` function for root finding.

Which of the keywords of `broyden()` do you need to tune to increase the precision of the result?

Now use `broyden()` to find a root of the system

$$\cos(x) + e^{-y} = 6 \tag{3}$$

$$\frac{x^3}{x^2 + y^2} = 2 \tag{4}$$

Verify that the numerical solution satisfies the equations (this is a one-liner in IDL).

Question 2 *Heat conduction in a rod*

Consider a thermally conducting thin rod of constant heat conductivity $\lambda = 1$. The left end of the rod ($x = 0$) is kept at constant temperature $T = 0$, the right end ($x = L \equiv \pi$) is thermally insulated, and along the rod a volume heating $q(x) = 2 + \cos(x)$ is applied.

(a) Find the steady state of the rod and plot temperature and heat flux as functions of x.

Hint: Use the fixed-time-step Runge–Kutta scheme `rk4.pro` to define a function $F(L; \varphi)$ that maps a guess φ for the heat flux $F(0)$ to a value at the right end. Then use `broyden()` to find the correct value of φ.

(b) Replace $q(x)$ by a narrow Gaussian centred somewhere near the middle of the rod. Explain what you get now.