Phys 535 **Assignment 9**

Deadline: Friday 2 December 2005

Question 1 *Advanced heat conduction with high-order method*

Solve the heat conduction equation

$$c_v \varrho(x,t) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(x,t;T)\frac{\partial T}{\partial x}\right] + q(x,t) \ ,$$

on the periodic interval $0 \le x < 2\pi$, where $c_v = 1$ is specific heat, $\varrho = 2 + \cos x$ is density, $\lambda(x,t;T) = 2 + 4T^2/(1+T^2) + \sin x$ is the heat conductivity, and $q(x,t) = \sin x \sin^3 \omega t$ is external heating with $\omega = 0.5$. The initial condition is

$$T(x,0) = |\cos 3x| \ .$$

Use a high-order method.

Note: You need to extract the part with the second derivative on the right-hand side – two first derivatives will make the code unstable.

Question 2 *Von Neumann stability analysis*

Carry out von Neumann stability analysis for the upwind scheme.

(a) Derive an expression for the amplification factor A as a function of Courant number $\mathcal{C}$ and dimensionless wave number $\kappa \equiv k\delta x$.

(b) Calculate the square of the complex modulus, $|A|^2$.

(c) Identify the most unstable wave number and find the critical $\mathcal{C}$ for this wave number.

Question 3 *Upwind scheme*

Use the upwind scheme to solve the advection problem

$$\frac{\partial f}{\partial t} = -u\frac{\partial f}{\partial x} \tag{1}$$

on the periodic interval $0 \le x < 2\pi$, where the advection velocity $u = 1$ is constant. The initial condition is

$$f(x,0) = \tanh(7\sin x) \ . \tag{2}$$

Note: Use IDL's `shift()` function to get f^l_{k-1} from f^l_k; it will do exactly the right thing for periodic values.

Question 4 *Advection with high-order scheme*

Use a high-order scheme to solve Eq. (1) with initial condition (2) for $u = 1 + \frac{1}{2}\cos 2\pi(x - 0.2)$ Run for 10 periods T_P, where $T_\mathrm{P} = 2\sqrt{3}/3 \approx 1.15$.

(a) Compare the final profile to the initial one (they are identical for the exact solution).

(b) Find good values for the time step and artificial diffusivity.

(c) Find the Courant time step (the value separating stable from unstable scheme).

(d) Is the spectral scheme of much help compared to 6th-order spatial derivatives?