Phys 535 **Exercises**

Deadline: Friday 25 November 2005

Question 1 *Spectral method in 3-d*

Use the spectral method to solve the Laplace equation

$$\Delta\Phi = 4\pi G\varrho$$

in three dimensions for a point mass M, i.e. a density ϱ that is $M/(8\,\delta x\,\delta y\,\delta z)$ for the 8 grid points closest to the centre $\mathbf{x} = \mathbf{0}$, and $\varrho = 0$ for all other points. Assume periodic boundary conditions for the gravitational potential in all directions. Use 128 grid points in each direction and use units where $GM = 1$ (G being Newton's gravity constant).

(a) Construct a three-dimensional grid and calculate ϱ on that grid.

(b) Apply the three-dimensional forward Fourier transform `fft(f,-1)` to the right hand side $4\pi G\varrho$ (if f is a three-dimensional array, `fft(f)` will automatically Fourier-transform in all three directions).

(c) Construct the wave number grids k_x, k_y, k_z and combine them into one wave vector $\mathbf{k}$.

(d) Construct an array `k_2` that is equal to $1/|\mathbf{k}|^2$ for $|k| \neq 0$ and is 0 for $k = 0$, etc.

(e) Plot the potential $\Phi(r)$ over spherical radius r along a few lines: coordinate, face diagonal, space diagonal, and compare to Newton's potential for an isolated point mass. Explain the difference.

(f) Can you also plot the modulus of the gravitational acceleration $\mathbf{g} = -\nabla\Phi$? Hint: what becomes of the gradient in Fourier space?

Question 2 *Heat conduction*

Solve the time-dependent heat conduction equation

$$\frac{\partial T}{\partial t} = \chi\frac{\partial^2 T}{\partial x^2} + q(x,y)$$

in one (spatial) dimension on the interval $0 \leq x \leq 1$ Use

$$q(x,t) = \sin(\omega t)\frac{e^{-(x-x_0)^2/(2w^2)}}{\sqrt{2\pi w^2}} \ ,$$

with $\chi = 1$, $\omega = \pi$, $x_0 = 2/3$, $w = 0.03$, the initial condition

$$T(x,0) = 0 \ ,$$

and the boundary conditions

$$T(0,t) = 0 \; , \qquad \frac{\partial T}{\partial x}(1,t) = 0 \; .$$

Use $N_x = 128$ points.

(a) Use the explicit scheme. Choose an appropriate time step δt (such that the results do not change visibly when you half δt) and overplot profiles $T(x, t_i)$ for some times $t_i \in \{0, 1/2, 1, 3/2, 2, 5/2, 3\}$

(b) What happens if you use a large time step? What is the critical time step, i.e. the step δt_* such that the scheme is stable for $\delta t < \delta t_*$ and unstable for $\delta t > \delta t_*$?

(c) Use the fully implicit scheme. What happens if you use a large time step (say 10 or 30 times δt_* from above)? Compare to the results plotted in (a).

(d) Use the Crank-Nicholson scheme. Find an appropriate time step in the same sense as for the explicit scheme and compare the two values.