Phys 535 **Exercises**

Deadline: Friday 28 October 2005

Question 1 *Random walk in 1 dimension*

A particle starts at $x = 0$ for $t = 0$ and then moves by Δ during the time interval δt. The values of Δ for different time steps are stochastically independent.

We expect the probability density function of the x-position X to spread according to the diffusion equation

$$\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial t^2} ,$$

which has the Green's function

$$G(x-x', t-t') = \frac{1}{\sqrt{4\pi D(t-t')}} \, e^{-\frac{(x-x')^2}{4D(t-t')}} \qquad (1)$$

(a) Assume $\Delta = \pm 1$, where the sign is randomly chosen for each time step. Plot the probability density function after $N_t = 1000$ steps and try to fit a Gaussian (1) to it. What is the diffusion constant D? How can you test whether the broadening of the PDF is really described by Eq. (1)?

(b) Repeat part (a) for $\Delta \sim \mathcal{U}(-0.5, 0.5)$.

(c) Returning to $\Delta = \pm 1$, try to obtain the PDF of the *return time*, i.e. the time it takes for one particle to return to the origin for the first time. Will the average return time be finite?

Question 2 *Two-dimensional Ising model*

Program the two-dimensional Ising model

$$E = -\varepsilon \sum_{i=0}^{N-1} s_{ik} (s_{i-1,k} + s_{i+1,k} + s_{i,k-1} + s_{i,k+1})$$

for $N \times N$ spins. Assume periodic boundary conditions.

(a) Vectorize the problem.

(b) Using $N = 20$ points in each direction, plot energy per spin E/N and magnetization per spin as function of temperature T.

(c) Plot the specific heat c_v as a function of temperature T.

(d) Do you see indications of a phase transition?

Hints:

- You may need to increase N, but keep in mind that equilibration will take longer then.
- The IDL function `shift(`*array*,n_1,n_2`)` takes an array and performs a cyclic shift of its elements by n_1 in the first direction and n_2 in the second.

Question 3 *The travelling salesman problem*

A salesman starts from Calgary to a round trip through the following Canadian cities:

- Edmonton
- Halifax
- Montreal
- Ottawa
- Vancouver
- (Calgary)

(a) Use *simulated annealing* to find a sequence of these cities that tries to minimize the total mileage.

(b) Increase the cooling 'time' and repeat the experiment to make sure you get consistent results.

(c) How long is the round trip? Sketch the route.

(d) Now assume that the Transcanada Highway is impassable between Calgary and Vancouver. How does the route change? [Assume that the other distances have not changed – which is quite inconsistent]

Hints:

- Number the cities and construct a distance matrix You can find the distances at `http://www.trailcanada.com/travel/gettingaround-distances.asp` or `http://www.craigmarlatt.com/canada/geography&maps/distances_between_cities.html`.
- Change the current route by interchanging an randomly picked pair of cities.