Deadline: Friday 21 October 2005

Question 1 *Monte Carlo integration*

Consider the three cylinders

$$\begin{aligned} \mathcal{C}_x &: \quad y^2 + z^2 \le 1 \ , \\ \mathcal{C}_y &: \quad x^2 + z^2 \le 1 \ , \\ \mathcal{C}_z &: \quad x^2 + y^2 \le 1 \ . \end{aligned}$$

Their intersections are called *Steinmetz solids*, in particular the *bicylinder*,

$$\mathcal{B} = \mathcal{C}_x \cap \mathcal{C}_y \ ,$$

and (what I here call) the *tricylinder*

$$\mathcal{T} = \mathcal{C}_x \cap \mathcal{C}_y \cap \mathcal{C}_z \ .$$

[See e.g. `http://astronomy.swin.edu.au/~pbourke/geometry/planelev`, or `http://mathworld.wolfram.com/SteinmetzSolid.html`]

(a) Find an upper estimate for the volume $||\mathcal{B}||$ and interpret it geometrically

(b) Write a program to get the volume $||\mathcal{B}||$ by Monte Carlo integration.

(c) Find $||\mathcal{T}||$ the same way. How many points does one need to get reasonable accuracy?

(d) Use Monte Carlo integration to determine the moment of inertia tensor J_{ik} for $\mathcal{T}$,

$$J_{ik} = \int \left(r^2 \delta_{ik} - x_i x_k \right) \, dm \ ,$$

assuming a density of 1. Interpret your result.

Question 2 *Geometric probability: Bertrand's problem*

Use two different methods to calculate the average length of the chord marked by a random straight line intersecting the unit circle. [Of course, do this numerically.]

1. Choose two random points on the circumference of the circle (i.e. choose their azimuth angles to be uniformly distributed) and use the straight line connecting the two.

2. Orient a radial line randomly [do you really need that?], choose a point P on it such that its radius (distance from the origin) is uniformly distributed on $[0, 1]$. Then draw the line through P, perpendicular to the radial direction.

Compare the two averages and try to discuss.