Phys 535 **Exercises**

Deadline: Friday 14 October 2005

Question 1 *Hat problem*

At a dinner party each of the N guests leaves his hat in the parlour. The fondue is so terrible that they all flee in a haste, each grabbing a hat at random. How large is the probability that at least one guest returns home (still pale with horror, but) with his own hat?

Solve this numerically for $N = 2, 3, 5, 10, 20$.

Question 2 *The Monty Hall problem*

In a game show, the candidate tries to win a goat (☺) that is hidden behind one of three doors, while the other two doors each hide a car (☹): The candidate first picks one of the doors, after which the compère opens one of the two remaining doors to show that there is a car behind it. Then the candidate has the opportunity to reconsider her decision and choose the other remaining door.

In order to see which strategy is better, simulate a sufficiently large number of games, one set where the player sticks with her original decision, and one where she changes her mind and chooses the other door.

What is the winning probability in both scenarios? Can you explain?

Question 3 *Exponential distribution*

(a) Write a function `randomexp()` that returns an array of (pseudo) random numbers x_i that are distributed according to the exponential distribution,

$$f_X(x) = \alpha e^{-\alpha x} \ . \tag{1}$$

(b) For $\alpha = 1$ and $\alpha = 10$, determine the average and the standard deviation; derive the analytical values of EX and $V(X)$ and compare.

(c) Plot a histogram of the PDE.

(d) Generate two other sets of random numbers y_i and z_i with the same distribution, and plot the histograms of $x_i + y_i$ and of $x_i + y_i + z_i$. Can you guess the functional form of $f_X(x)$ for the two cases? Explain the result in the light of the interpretation of the exponential and gamma distributions.

Question 4 *Cauchy distribution, central limit theorem*

(a) Write a function `randomcauchy()` that returns an array of (pseudo) random numbers x_i that are distributed according to the Cauchy distribution.

(b) Determine the average and the standard deviation and plot a histogram of the PDE. Does the average become more accurate if you increase the sample count?

(c) Now plot the PDE for the sum of 100 random numbers for each of the distributions

1. uniform
2. exponential
3. normal
4. Cauchy

Marvel and discuss.

Question 5 *Acceptance-rejection method*

Use the acceptance-rejection method to generate random numbers X for Student's z-distribution with $n = 4$,

$$f_X(x) = \frac{2}{\pi} \frac{1}{(1 + x^2)^2} \ . \tag{2}$$

1. Use the Cauchy profile as comparison function. What is the amplitude factor you need?
2. Plot a histogram of the PDE, determine mean and variance.
3. What is the efficiency η (i.e. the fraction of accepted pairs)?