Exercise 1 *The* `atanh` *function*

There is no `atanh` function in Fortran for the inverse hyperbolic tangent. Change this.

Exercise 2 *The factorial function using recursion*

(a) Write a Fortran function that recursively computes the factorial $n!$ of a non-negative integer n.

(b) Use that function to tabulate $n!$ for $n = 0..22$.

Exercise 3 *Complex numbers – polar representation*

Using only real arithmetic and the `atan` function,

(a) write a F90 subroutine that converts the real and imaginary parts x and y of a complex number z to modulus r and phase (argument) φ:

$$z \;=\; x + \mathrm{i}\, y \;=\; r\, \mathrm{e}^{\mathrm{i}\varphi} \;.$$

(b) Test your subroutine on 1, i, -1+i, 1+i, 1$-$i, $-1-$i. If φ is incorrect, fix the subroutine using `if`–`then`–`else`.

(c) Write another subroutine for the same task using the `atan2` function.

Exercise 4 *Celestial mechanics*

(a) Write a program that asks for mass M and radius R of a planet and prints out its first and second cosmic speeds v_1, v_2.

(b) Improve your program structurally by putting the calculation of the cosmic speeds into a subroutine.

(c) Print a table of v_1 and v_2 for all planets of our solar system, using the following data:

```
name   = (/'Mercury', 'Venus', 'Earth', 'Mars', 'Jupiter', 'Saturn', &
              'Uranus', 'Neptune', 'Pluto', 'Sedna' /)
mass   = (/ 0.330,     4.87,    5.97,    0.642,  1899,      568      &
              86.8,     102,       0.0125,  0.004  /) ! unit: 1E24 kg
radius = (/ 2.44,      6.052,   6.378,   3.397,  0.071492,  60.268,  &
              25.559,   24.764,    1.195,   0.8    /) ! unit: 1E6 m
```