Phys 499 **Exercises**

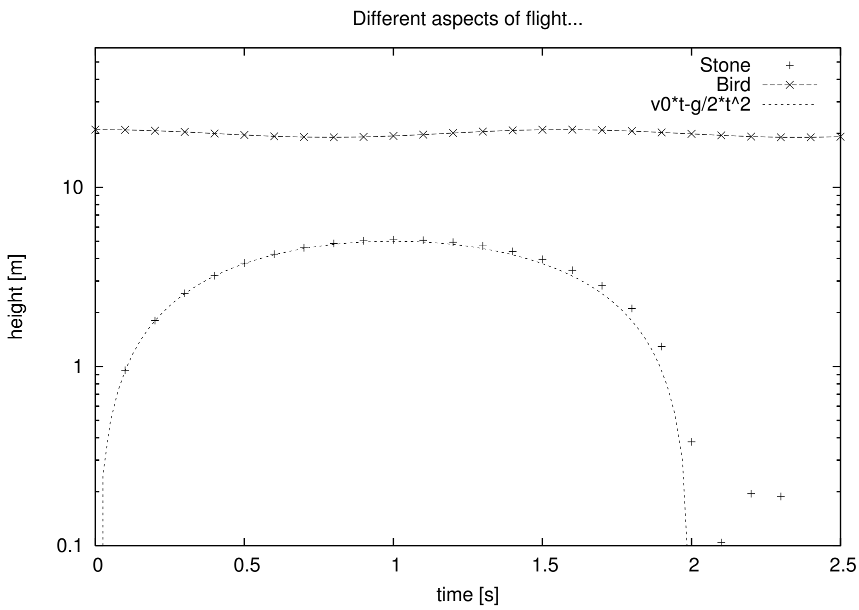
Deadline: Friday 4 February 2005

Question 1 *Plotting data from a file*

Get the file '`height.dat`' from `http://www.capca.ucalgary.ca/~wdobler/teaching/phys499/programs/height.dat`.

(a) Use gnuplot to plot h_stone(t) and h_bird(t) in one single graph.

(b) Write a gnuplot script to reproduce the following figure:



(c) Produce a PostScript plot of your graph and print it out.

Question 2 *The Feigenbaum function*

In the interval $x \in [-1, 1]$, the universal Feigenbaum function $g(x)$ can be approximated by

$$g(x) = a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + \dots$$

with

$$\begin{aligned} a_0 &= 1\,, \\ a_2 &= -1.5276329970\,, \\ a_4 &= 0.1048151948\,, \\ a_6 &= 0.0267056705\,, \\ a_8 &= -0.0035274096\,, \\ a_{10} &= 0.00008160097\,, \\ a_{12} &= 0.00002528508\,, \\ a_{14} &= -2.55632 \times 10^{-6}\,. \end{aligned}$$

For $|x| > 1$, the functional relation

$$g(x) = -\alpha\, g[g(x/\alpha)]$$

can be used to map the argument nearer to $x \in [-1, 1]$. Here

$$\alpha = 2.502907875096\ldots$$

is the Feigenbaum *reduction parameter*.

(a) Write a recursive F90 function that calculates $g(x)$.

(b) Embed this function as internal function into a main program that tabulates $g(x)$ for $-30 \geq x \geq 30$ and redirect the output to a file.

(c) Use *Gnuplot* to plot $g(x)$ with isotropic axis scaling.

(d) Would it be easy to re-write your function avoiding recursion?

References:

- M. J. Feigenbaum, "Quantitative Universality for a Class of Non-Linear Transformations", *J. Stat. Phys.* **19**, 25–52 (1978).
- `http://mathworld.wolfram.com/FeigenbaumFunction.html`

Question 3 *Allocatable and assumed-shape arrays*

(a) Write a program that reads an integer n, then allocates and sets the values for the Hilbert matrix $\mathbf{H}_n$ of order n,

$$\mathbf{H}_n = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots & \frac{1}{n+1} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \cdots & \frac{1}{n+2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \frac{1}{n+2} & \frac{1}{n+3} & \cdots & \frac{1}{2n-1} \end{pmatrix} .$$

Print out $\mathbf{H}_4$.

Hint: Use an allocatable array.

(b) Write a subroutine `flip(h)` that flips the matrix horizontally and apply it to $\mathbf{H}_n$ to get

$$\begin{pmatrix} \frac{1}{n} & \cdots & \frac{1}{3} & \frac{1}{2} & 1 \\ \frac{1}{n+1} & \cdots & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{n+2} & \cdots & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} \\ \vdots & \iddots & \vdots & \vdots & \vdots \\ \frac{1}{2n-1} & \cdots & \frac{1}{n+2} & \frac{1}{n+1} & \frac{1}{n} \end{pmatrix} .$$