Deadline: Friday 21 January 2005

Question 1 *Relativistic kinetic energy*

According to special relativity, the kinetic energy of a body of mass m moving at speed v is

$$E_{\text{kin}} = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) , \tag{1}$$

where $c \approx 3 \times 10^8\,\text{m/s}$ is the speed of light.

In the classical limit, Eq. (1) becomes

$$E_{\text{kin}} = \frac{m}{2} v^2 \left[1 + O\left(\frac{v^2}{c^2} \right) \right] \tag{2}$$

(a) What is the precision of your favourite calculator? A simple test is to calculate

$$100/81 = 1.2345679012345679012345679012345679012345679012345679\ldots$$

and subtract 1.23456790 (maybe including more digits) to see where the calculator's result starts to deviate from the exact sequence.

(b) Use your favourite calculator (the same one as above) and Eqs. (1) and (2) to calculate the kinetic energy of a car (mass $m = 1000\,\text{kg}$) that moves at a speed of $25\,\text{m/s}$.

Which of the formulas gives the correct results? Why?

(c) Along the lines of what we did in the first hour with the expression $1 - \sqrt{1 - \varepsilon}$, rewrite Eq. (1) in a form that is numerically better behaved, and evealuate E_{kin} for the car using that formula.

(d) Apply Eqs. (1), (2), and your improved formula to a particle (mass $m = 1.67 \times 10^{-27}\,\text{kg}$) at $v = 2.9 \times 10^8\,\text{m/s}$ and discuss the results.