A New Mechanism of Nonlinearity in the Disc Dynamo

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Abstract

We propose and discuss a new mechanism for the saturation of magnetic field growth for mean-field dynamos in galactic and accretion discs. A peculiar property of these dynamo systems is that the disc thickness grows with the magnetic field. The efficiency of magnetic field regeneration can be shown to have a maximum at a moderate value of the disc half-thickness h (for the Solar vicinity of the Milky Way, at about 600 pc). Therefore, at small h the dynamo action gets stronger with h and the magnetic field may grow super-exponentially. The disc thickness eventually becomes so large that the dynamo action is suppressed and a steady state is established. By comparing this nonlinearity with α-quenching we show that the latter is more efficient in establishing the steady-state strength of the mean magnetic field in real spiral galaxies. The disc inflation caused by the growing magnetic field may be important for the history of magnetic field evolution in galaxies; this effect can strongly accelerate the generation of the regular magnetic field in young galaxies.

1 Introduction

The turbulent mean-field dynamo [1] is generally regarded as the principal source of large-scale magnetic fields in spiral galaxies and accretion discs [2, 3]. The linear stage of magnetic field growth, in which the response of the fluid motion due to the growing magnetic field can be neglected, is rather well understood and interest is now focussed mainly on nonlinear dynamo effects which determine the stationary or quasi-stationary states corresponding to the observable magnetic field structures. A self-consistent treatment of the nonlinear dynamo problem assumes a simultaneous solution of the induction equation and the Navier-Stokes equation for a turbulent velocity field — a task mostly far beyond the scope of the modern theory. To get some insight into the nonlinear behaviour of dynamos, a phenomenological approach is often used in which certain (simple) nonlinearities are introduced into the induction equation (usually, suitably averaged). For the turbulent mean-field dynamo, several such mechanisms of nonlinearity have been investigated, which are mostly connected with the action of magnetic force on the velocity field.

The dominant nonlinearity in the mean-field dynamo equations, as applied to galaxies and stars, is often assumed to be due to the influence of the magnetic field on the turbulent component of the fluid velocity. The generation of the mean magnetic field in turbulent flows is possible as far as the mirror symmetry of the velocity field is broken [1, 2]; the violation of mirror symmetry is quantified by an additional term in the induction equation for the mean field (known as the dynamo equation), containing the so-called α-coefficient which is in the simplest case proportional to the mean helicity of the turbulence. The influence of the magnetic field on the turbulent flow is then supposed to lead to a reduction ("quenching") of the α-coefficient so that the growing magnetic field reduces the efficiency of its generation and a stationary state is eventually established [1, 4, 5, 6]. Steady-state dynamo regimes associated with the action of the Lorentz force on the large-scale velocity have also been considered [7, 8, 9, 10, 11], and likewise those resulting from magnetic buoyancy effects.

In the present paper we discuss another mechanism of nonlinearity for the mean-field dynamo which has not been examined until now and is not associated with a modification of the velocity field by the Lorentz force. This mechanism is specific for astrophysical gas discs that do not have fixed rigid boundaries, e.g. the gaseous discs of spiral galaxies and accretion discs. The thickness of such a disc is determined by the balance between the gradient of the total pressure (including magnetic pressure) and the gravitational force that is due to stars in the case of galaxies and to the central massive object in the case of accretion discs. It is thus clear that the disc thickness $2h$ grows when the magnetic field strength increases because of the dynamo action. Insofar as the magnetic field contribution to the total pressure [2] in astrophysical discs is significant, this variation of h can be essential. The efficiency of magnetic field generation changes with h not monotonically but it increases with h for moderate values of the disc thickness and decreases for large h. As a result, this nonlinearity leads to interesting consequences: it first accelerates the growth of the magnetic field and then can suppress it at later stages.

It is only natural to ask whether this mechanism might be of importance in solar and stellar dynamos. On the one hand, these objects also do not have a fixed boundary (contrary to, e.g., the Earth's liquid core surrounded by the mantle), and the plasma is compressible. Thus it should be expected that the size of the dynamo system depends, to a certain extent, on the magnetic pressure. On the other hand it should be kept in mind that the thickness of the convective zone, in which the dynamo works, is determined not by magnetic pressure, but by the temperature distribution along radius, heat transport, etc.

Below we discuss in detail a saturation mechanism for the magnetic field in astrophysical discs inflated by magnetic pressure. This effect may have important consequences, like a super-exponential field growth at early stages of dynamo action. We will show, however, that in real galaxies α-quenching is more important in limiting the magnetic field growth and the steady-state field strength is, as a rule, determined just by α-quenching.

Although our basic results are applicable to both galactic and accretion discs, numerical values of the parameters used below always refer to the solar neighbourhood in our Galaxy.

2 The effect of the disc thickness on the dynamo

The generation of magnetic field in a turbulent flow with broken mirror symmetry is described by the well-known mean-field dynamo equation [1, 2, 12, 13]

$$\frac{\partial \mathbf{B}}{\partial t} = R_\alpha \nabla \times (\alpha \mathbf{B}) + R_\omega \nabla \times (\mathbf{V} \times \mathbf{B}) + \Delta \mathbf{B} \,, \tag{1}$$

written in a standard dimensionless form with distance, measured in units of the disc half-thickness, and time, in units of the associated magnetic diffusion time. $\mathbf{B}$ is the mean magnetic field, α is the α-coefficient characterizing the violation of mirror symmetry in the turbulent flow, $\mathbf{V}$ is the mean velocity field (in astrophysical discs, this is usually differential rotation $\mathbf{V} = \omega \times \mathbf{r}$). Finally, R_α and R_ω are the turbulent magnetic Reynolds numbers for the α-effect and differential rotation, respectively, characterizing the intensities of these generation mechanisms.

Equation (1) is supplemented with boundary conditions posed, for instance, at the disc surface, $|z| = h$, $r = R$. Since the disc half-thickness h is considered as a function of $\mathbf{B}$, the nonlinearity in our model is introduced through the boundary conditions (in addition to other, conventional nonlinearities). We use cylindrical coordinates with the origin at the disc centre and the z-axis parallel to the rotation axis.

In the framework of the widely used $\alpha\omega$-dynamo approximation [1, 2, 3, 12, 13], solutions depend not on the magnetic Reynolds numbers R_α and R_ω individually, but on their product $D \equiv R_\alpha R_\omega$, the so-called dynamo number that can be expressed in terms of the mean and turbulent flow parameters in the following way (see, e.g., [14]):

$$D \simeq 10 \left(\frac{\omega h}{v} \right)^2 \,. \tag{2}$$

Here v is the root-mean-square turbulent velocity, ω the angular velocity and h the half-thickness of the disc in which the magnetic field is generated. We emphasize that D strongly depends on h. [To obtain Eq. (2), we have used the order-of-magnitude estimates $\alpha \simeq l^2\omega/h$ and $\beta \simeq \frac{1}{3} l v$, with l the turbulent scale and β the turbulent diffusivity.]

Any dynamo mechanism is of a threshold character, so that the magnetic field can grow only if the dynamo number exceeds a certain critical value $D_{\rm cr}$. In the case of a thin disc, $D_{\rm cr}$ has a value between 6 and 10, depending on the distribution of α across the disc. As follows from Eq. (2), for $\omega \approx 10^{-15}\,\mathrm{s}^{-1}$ and $v \approx 10\,\mathrm{km\,s}^{-1}$ (values typical of the galactic neighbourhood of the Sun) the critical value of the dynamo number $D_{\rm cr} = 10(\omega h_{\rm cr}/v)^2$ equal to, say, 7 is reached for $h = h_{\rm cr} \approx 300\,\mathrm{pc}$, which is slightly smaller than $h_\odot \simeq 400$–$500\,\mathrm{pc}$, the observed half-thickness of the ionized gas disc in the solar neighbourhood (cf. [15] and [3], §VI.2).

The basic fact for our further discussion of the disc dynamo in an inflated disc is the non-monotonic dependence of the dynamo time scale τ on the disc thickness. We first show this at a heuristic level using asymptotic estimates for the local dynamo equations where the distance to the disc axis is assumed constant; these equations represent the first approximation to Eq. (1) for a thin disc [16, 3]. We use the fact that the nonlinearity associated with the boundary conditions enters the local dynamo equations (see Eq. (12) below) only through their coefficients, i.e. *parametrically*. Therefore, well-known asymptotic estimates for the growth rate and length scale of the magnetic field in the kinematic dynamo are still applicable. Our final conclusions are based on a numerical solution of nonlinear Cauchy problem for Eq. (1) in the thin-disc approximation, including α-quenching as well as the nonlinear boundary conditions.

The physical reason for the non-monotonic dependence of the instantaneous growth rate of the magnetic field γ on h consists in the fact that the field scales are very different from each other near the generation threshold and for large values of D. For moderate values of the dynamo number, the characteristic scale L_B of the large-scale magnetic field is close to the disc thickness and is independent of D, whereas for $D \gg D_{\rm cr}$ this scale depends on the dynamo number [13, 17].

According to the threshold nature of the dynamo mechanism, the growth rate γ increases linearly with D for $D \approx D_{\rm cr}$ (this can be easily seen from the local dynamo equations if one retains the terms of first order in the small parameter $D - D_{\rm cr}$). Thus, γ is estimated as (in dimensional units)

$$\gamma \simeq C_1 \frac{\beta}{h^2}(D - D_{\rm cr}) \quad \text{for} \quad \left|\frac{D}{D_{\rm cr}} - 1\right| \ll 1\,, \tag{3}$$

with C_1 a constant of order unity. Using Eq. (2), we obtain for $h \approx h_{\rm cr}$:

$$\gamma \simeq C_1 \frac{\beta}{h_{\rm cr}^2} D_{\rm cr} \left(\frac{h}{h_{\rm cr}} - 1\right) \quad \text{for} \quad \left|\frac{h}{h_{\rm cr}} - 1\right| \ll 1\,. \tag{4}$$

Hence, γ increases linearly with h near the generation threshold.

Let us now estimate γ for $D \gg D_{\rm cr}$. Assume that $\partial/\partial z = O(D^n)$, $\partial/\partial t = O(D^m)$, and $b_r = O(D^k b_\phi)$. Then the requirement that all the terms in the local dynamo equations (12) are of the same order of magnitude in D yields $m = 1/2$, $n = 1/4$ and $k = -1/2$ (see [13, 17] for details). Thus, the required asymptotic estimate for the growth rate takes the form (in dimensional units) [13, 17]: $\gamma \simeq D^{\frac{1}{2}}\beta/h^2$. Using Eq. (2), we find that for $h \gg h_{\rm cr}$ the growth rate decreases with h:

$$\gamma \simeq C_2 \frac{\beta}{h_{\rm cr}^2} D_{\rm cr}^{\frac{1}{2}} \frac{h_{\rm cr}}{h} \quad \text{for} \quad \frac{h}{h_{\rm cr}} \gg 1\,, \tag{5}$$

where $C_2 = O(1)$. (We assume here that the turbulent scale l and, consequently, β, are independent of h; a reasonably weak dependence $l \propto h^k$ with $k < 1$ is still admissible.)

In Fig. 1 we show these asymptotics together with the dependence of the local growth rate on the disc half-thickness obtained from a numerical solution of the kinematic $\alpha\omega$-dynamo model for the solar neighbourhood in our Galaxy (see §4 below). A dependence of this kind is rather typical of spiral galaxies. Note the good agreement between the asymptotic and numerical solutions, especially for $D \gg 1$. Below we also discuss numerical solutions of Eq. (1) with allowance for nonlinear effects.

3 The effect of the magnetic field on the disc thickness

The hydrostatic scale height of the galactic gaseous disc is established due to a balance of the gravitational force and the gradient of the total pressure $P = P_{\rm th} + P_{\rm turb} + P_{\rm mag} + P_{\rm cr}$. The contribution of thermal and turbulent pressure can be estimated as $P_{\rm th} + P_{\rm turb} = \varrho v_{\rm s}^2 + \frac{1}{3}\varrho v^2 \approx \varrho v^2$ (we assume the pressure of the chaotic magnetic field to be also included in this term). Here ϱ is the gas density and $v_{\rm s}$ is the isothermal sound speed. Making the standard assumption that the cosmic ray pressure equals the magnetic pressure, we have $P_{\rm mag} + P_{\rm cr} \approx B^2/4\pi$. A formula obtained by Parker ([2], §22.1.4) then follows under the assumption that all the components of the total pressure depend on z in the same way:

$$h = \frac{\varrho v^2 + B^2/4\pi}{\varrho \langle g \rangle} = h_0 \left(1 + \frac{B^2}{{B_h}^2} \right) , \tag{6}$$

where $\langle g \rangle$ is the gravitational acceleration averaged over the interval $0 \le z \le h$, $h_0 = v^2/\langle g \rangle$ is the half-thickness of the disc without the regular magnetic field, and B_h is a characteristic field strength, ${B_h}^2 = 4\pi\varrho v^2$.

Since the disc thickness is a function of time, it is instructive to express h not in terms of B and ϱ but via the variables $Q = Bh$ and $\sigma = \varrho h$, i.e. the magnetic flux per unit radial length and the surface gas density, respectively. The advantage of this is that these quantities are conserved when h varies with time and the magnetic field is frozen into the gas. In terms of these variables, Eq. (6) takes the form

$$h = \frac{1}{2} h_0 \left(1 + \sqrt{1 + \frac{Q^2}{\pi h_0 \sigma v^2}} \right) . \tag{7}$$

Equations (6) and (7) are applicable as far as $\langle g \rangle$ is independent of the height above the disc, which is the case outside the stellar disc, i.e. for $h \gtrsim 500\,{\rm pc}$. For a uniform distribution of the stellar density in z, we have $g \propto z$, so that $\langle g \rangle \propto h$ within the disc. Then the balance of gravity and pressure gradient yields [18]

$$h = h_0 \sqrt{1 + \frac{B^2}{{B_h}^2}} . \tag{8}$$

Using again the variables Q and σ, one can easily obtain a cubic equation for h.

Note that the h_0's in Eqs. (7) and (8) are different from each other. Equation (8) is applicable for small values of the disc thickness (e.g. for small B) and (7) or (6), for large h exceeding the stellar disc scale height.

In Eqs. (6) and (8), B, v and ϱ are actually average values of the magnetic field strength, turbulent velocity and density across the disc. As far as only the ratio $B^2/\varrho v^2$ enters Eqs. (6) and (8), it is possible to use the values at $z = 0$ instead, and we will do so.

We emphasize that the only difference between Eqs. (7) and (8) is that in the latter h grows slower with the magnetic field. However, only the fact that h is a monotonically growing function of Q is important for our discussion. As a result, the growth rate of the magnetic field first grows with B as $h - h_{\rm cr}$ [see Eq. (4)] and then declines as $1/h$ [see Eq. (5)].

According to Eq. (5), the growth rate γ remains positive for $h > h_{\rm cr}$, however large is h. This would mean that the growth of the magnetic field continues for any value of h and a steady state cannot be achieved. However, the above estimates are based on a one-dimensional model [i.e. Eqs. (12) below] and it is not difficult to see that even a weak radial magnetic diffusion leads to establishing a steady state. The steady-state strength of the regular magnetic field then follows from the balance between the local growth rate γ and the rate of the radial magnetic diffusion $\beta/\delta r^2$ (with δr the radial field scale) as

$$\left(\frac{B}{B_h} \right)^2 \simeq \frac{\delta r^2}{h_0 h_{\rm cr}} D_{\rm cr}^{\frac{1}{2}} \tag{9}$$

for Eq. (6). As an illustration, consider $\delta r = 2\,{\rm kpc}$, $h_0 = 0.2\,{\rm kpc}$ and $h_{\rm cr} = 0.3\,{\rm kpc}$. Then $B/B_h \simeq 15$. Adopting the standard estimates $\varrho \simeq 1.7 \times 10^{-24}\,{\rm g\,cm^{-3}}$ and $v \simeq 10\,{\rm km\,s^{-1}}$, we have $B_h \simeq 5\,\mu$G, which leads to a steady-state field of the order of $75\,\mu$G (we suppose here that h does not differ much from 400 pc in the stationary state). Using Eq. (8) instead of (7), we obtain an even higher field strength. The regular magnetic fields observed in spiral galaxies have a strength generally not higher than 2–3 μG. This indicates that in real galaxies the steady-state field strength is established by α-quenching (or some other nonlinear effect).

To explore the interplay of these two mechanisms of nonlinearity in the mean-field dynamo, we consider below numerical solutions for a thin-disc $\alpha\omega$-dynamo incorporating both the nonlinear boundary conditions and α-quenching.

4 The dynamo in an inflated thin disc

Consider the thin-disc approximation which is applicable as long as the disc thickness is much less than the distance from the rotation axis, $h/r \leq 0.1$ [3]. In that case, solutions of Eq. (1) can be represented as $\mathbf{B}(r,z) = Q(r)\mathbf{b}(z;r)$, where $\mathbf{b}$ depends on r only parametrically. For the sake of simplicity, we consider axisymmetric magnetic fields.

Basic equations of the thin-disc dynamo can be found in Refs. 3 and 14. Here we discuss only the modifications of the standard model associated with the time dependence of the disc thickness.

For $\mathbf{b} = (b_r, b_\varphi)$, the following dimensionless equations written in cylindrical coordinates result from Eq. (1) (z and r are measured in units of the characteristic disc half-thickness and radius, h_* and r_*, respectively, and unit time is h_*^2/β):

$$
\begin{aligned}
\frac{\partial b_r}{\partial t} &= \frac{\partial^2 b_r}{\partial z^2} - R_\alpha \frac{\partial}{\partial z}(\alpha b_\phi) - R_z \frac{\partial}{\partial z}(V_z b_r), \\
\frac{\partial b_\phi}{\partial t} &= \frac{\partial^2 b_\phi}{\partial z^2} + R_\omega G b_r - R_z \frac{\partial}{\partial z}(V_z b_\phi), \\
& b_r|_{z=\pm h} = b_\phi|_{z=\pm h} = 0,
\end{aligned}
\tag{10}
$$

where α, h, V_z, $\mathbf{b}$ and γ depend parametrically on r and Q. Here γ is the local growth rate of the magnetic field, and $G \equiv r\,d\omega/dr$ is the velocity shear (a function of r). In this approximation, $b_z = 0$. Although the coefficients R_α and R_ω enter the system (10) separately, the transformation $b_\phi \to R_\alpha b_\phi$ immediately shows that the solutions depend only on their product $D \equiv R_\alpha R_\omega$ (as for the dependence on R_z, see below).

Equations (10) differ from the standard local $\alpha\omega$-dynamo equations by the terms with the vertical velocity V_z which result from the inflation of the disc, $V_z|_{z=h} = \partial h/\partial t$. Accordingly, we introduce a new magnetic Reynolds number $R_z \equiv V_{z*}h_*/\beta$ with asterisk denoting a typical value. Below we adopt a simple z-dependence of the vertical velocity: $V_z = z\dot{h}/h$ where dot denotes time derivative. As we will show now, the solutions of Eq. (10) depend on V_z in a physically insignificant manner because V_z leads only to a rescaling of the vertical axis.

In order to transform Eq. (10) into the standard form, we introduce new variables

$$
\tilde{z} = \frac{z}{h}, \quad \tilde{b}_r = b_r h, \quad \tilde{b}_\phi = R_\alpha b_\phi h. \tag{11}
$$

Transforming partial time derivatives using the relation $(\partial\tilde{z}/\partial t)|_{z=\mathrm{const}} = -\tilde{z}\dot{h}/h$, we obtain, after replacing $(\partial/\partial t)|_{\tilde{z}=\mathrm{const}}$ by γ:

$$
\begin{aligned}
h^2\gamma\tilde{b}_r &= \frac{\partial^2\tilde{b}_r}{\partial\tilde{z}^2} - h\frac{\partial}{\partial\tilde{z}}(\alpha\tilde{b}_\phi), \\
h^2\gamma\tilde{b}_\phi &= \frac{\partial^2\tilde{b}_\phi}{\partial\tilde{z}^2} + h^2 D\tilde{b}_r, \\
& \tilde{b}_r|_{z=\pm 1} = \tilde{b}_\phi|_{z=\pm 1} = 0.
\end{aligned}
\tag{12}
$$

Thus, the local boundary value problem reduces to the standard equations of the $\alpha\omega$-dynamo in a slab [13, 2, 3] if we introduce $\tilde{\alpha} \equiv h\alpha$ and $\tilde{D} \equiv h^2 D$. Since $\alpha \simeq l^2\omega/h$, the new coefficient $\tilde{\alpha}$ does not depend on h, and $\tilde{D}$ is the effective dynamo number defined to incorporate the time variation of the disc thickness.

The physical meaning of the variables (11) is as follows. The terms including V_z in (10), $\partial(V_z\mathbf{b})/\partial z = V_z\partial\mathbf{b}/\partial z + \mathbf{b}\partial V_z/\partial z$, describe the advection of the magnetic field in the z-direction (the first term) and the dilution of the magnetic field in the inflated disc associated with the conservation of magnetic flux (the second term). Introducing the Lagrangian variable $\tilde{z}$ we get rid of the first term, and introducing $\tilde{\mathbf{b}}$ we explicitly make use of the fact that magnetic flux is not affected by a mere inflation of the disc.

The specific form of the new variables $\tilde{z}$, $\tilde{\mathbf{b}}$ depends on the dependence of V_z on z adopted and has been given above for $V_z \propto z$. For an arbitrary function $V_z(z)$, the new variables are to be defined in the following way:

$$\tilde{z} = \frac{1}{h|_{t=0}}\left(z - \int_0^t V_z|_{\tilde{z}=\text{const}}\, dt\right), \quad \begin{pmatrix}\tilde{b}_r \\ \tilde{b}_\phi\end{pmatrix} = \begin{pmatrix} b_r \\ R_\alpha b_\phi\end{pmatrix} \cdot \left.\frac{\partial z}{\partial \tilde{z}}\right|_{t=\text{const}}. \tag{13}$$

It is convenient to normalize the eigenfunction $\mathbf{b}$ of the boundary value problem (12) as $\int_{-1}^{1}(\tilde{b}_r^2 + \tilde{b}_\phi^2 R_\alpha^{-2})d\tilde{z} = 1$. This normalization ensures that the ratio Q/h is a characteristic strength of the magnetic field at distance r from the axis, $B(r) \approx Q/h$. Note that this physical meaning of the variable Q is somewhat different from that in the standard thin-disc formulation [3].

Arguments completely similar to those in Ref. 14 lead to the following equation for Q:

$$\begin{gathered}\frac{\partial Q}{\partial t} - \gamma\left(1 - \frac{Q^2}{h^2 {B_\alpha}^2}\right) Q - \lambda^2 \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rQ)\right] = 0, \\ Q(0) = Q(R) = 0,\end{gathered} \tag{14}$$

where $\lambda = h_*/r_*$ is the small parameter of the thin disc ($\lambda \simeq 0.05$) and B_α represents a characteristic magnetic field strength at which α-quenching becomes important. The local growth rate γ entering Eq. (14) is simply related to the eigenvalue of the system (12) that depends *parametrically* on r and Q. The nonlinear term on the right-hand side of Eq. (14) has been derived in Ref. 14 and describes α-quenching. To obtain Eq. (14), it was assumed that the α-coefficient depends only on the magnetic field averaged along the z-direction.

As shown in Ref. 14, $B_\alpha \simeq 2\,\mu$G, i.e. B_α and B_h are of the same order of magnitude. It can be seen from Eq. (14) that the growth of the magnetic field saturates for $B \to B_\alpha$ and α-quenching establishes a steady state with $B \simeq B_\alpha$, i.e. at a magnetic field strength much smaller than the stationary state due to nonlinear boundary conditions — see Eq. (9).

Strictly speaking, the introduction of the Lagrangian variable $\tilde{z}$ in the local equations (12) leads to a certain additional term in the radial equation (14). We neglect this term, since it arises because the disc is not flat (i.e. h is a function of r) and is significant only far from the disc axis where magnetic field strength is negligibly small.

5 Results

We have examined numerical solutions of the nonlinear Cauchy problem (14) adopting the initial condition $Q|_{t=0} = 10^{-3}$ with a small correction to meet the boundary condition $Q|_{r=0} = 0$. The generation of magnetic field has been investigated taking the example of our Galaxy. We adopted the rotation curve given by Clemens [19] and the initial disc thickness of the form $h_0 = 140\,\text{pc} \times \sqrt{1 + (r/4\,\text{kpc})^2}$, i.e. h_0 at $r = 0$ was chosen to be somewhat smaller than the present-day value $h_0|_{r=0} \approx 150$–$190\,\mathrm{pc}$ (cf. [3]). In Eq. (12), the parameters D and α can be expressed in terms of observable parameters of the disc. Having adopted a specific model of the Galaxy we obtain γ for every r and Q by solving the eigenvalue problem (12) numerically and then solve, also numerically, the Cauchy problem for the nonlinear equation (14) (see Ref. 3 for details).

The parameters B_h and B_α are proportional to $\sqrt{\varrho}$ and are chosen to depend on r both in the same way, so that their ratio is independent of r. The observed density distribution of the interstellar gas in the Galaxy is given, e.g., in Ref. 20. We have used this distribution, multiplied by a factor $h(r,0)/h(r,t)$ to account for mass conservation in an inflated disc. As it was not our aim to construct a very realistic dynamo model for the Galaxy, the numerical results below should be regarded only as illustrative examples.

The local rate of exponential growth of the magnetic field in the solar neighbourhood, γ, is shown as a function of h in Fig. 1 as obtained from the numerical solution of Eq. (12). The efficiency of the dynamo grows with the disc thickness as long as $h_\odot \lesssim 600\,\mathrm{pc}$, with $h_\odot$ denoting the half-thickness of the ionized galactic disc in the solar neighbourhood — and then decreases. Note that γ reaches a maximum for $h_\odot$ not much higher than the observed value, so in real galaxies both regimes of the nonlinear influence of the disc thickness on the dynamo can be realized: enhancement as well as quenching.

This can be clearly seen in Figs. 2 and 3 where we show how the instantaneous growth rate of the magnetic field $1/\tau \equiv d(\ln B)/dt$ varies with time for different ratios of B_h to B_α (exponential growth corresponds to $d(\ln B)/dt = \mathrm{const}$). As far as the values of B_h and B_α given above are merely order-of-magnitude estimates, it makes sense to consider a certain range of values for their ratio. In Fig. 2, we show the results when h depends on B as given in Eq. (6). The three curves shown here correspond to different values of B_h/B_α.

At the linear stage of the dynamo, all three curves behave identically: $1/\tau$ first decreases because the initial field is far from being an eigenfunction, and then starts the phase of exponential growth. The monotonically decreasing, dotted curve corresponds to $B_h/B_\alpha = 10$, in which case α-quenching is the dominant mechanism of nonlinearity. Then the disc thickness changes very little when the magnetic field grows and the growth rate monotonically tends to zero. At $t \approx 7.5 \times 10^9\,\mathrm{yr}$, the magnetic field growth is decelerated to such an extent that the characteristic time of its change reaches $5 \times 10^9\,\mathrm{yr}$, a state that can be regarded as stationary.

The thin solid line represents the case $B_h/B_\alpha = 1$. Now the growing magnetic field substantially inflates the disc, and the dynamo number grows significantly. Hence, B first grows faster than in the former case. However, when the disc thickness exceeds a certain value, further disc inflation hampers the field growth and thus the second curve declines faster than the first one; note also that α-quenching is facilitated by a more rapid early growth of the magnetic field.

Finally, the thick solid line shows the case $B_h/B_\alpha = 0.5$ and it can be seen that for $t \lesssim 5 \times 10^9\,\mathrm{yr}$ the effect of the disc inflation on the dynamo is stronger than that of α-quenching. Initially, the disc inflates strongly, causing the magnetic field to grow super-exponentially. Then α-quenching soon becomes important and the field approaches its steady state quite quickly, reaching it already for $t \approx 6.5 \times 10^9\,\mathrm{yr}$.

Fig. 3 shows similar results for the nonlinearity stemming from Eq. (8). The influence of this nonlinearity on the dynamo efficiency is not as strong as in Fig. 2 because now h depends on B weaker than in Eq. (7). However, the magnetic field evolution is qualitatively the same: our conclusions are not very sensitive to the chosen form of the dependence $h(B)$.

Thus, the investigated nonlinearity in the dynamo equations, associated with the dependence of the disc thickness on the magnetic pressure, enhances the dynamo action — possibly even leading to a super-exponential growth of the magnetic field — and substantially accelerates the establishing of the steady state. Yet, other mechanisms of nonlinearity like α-quenching play a major role in the saturation of the dynamo instability.

6 Discussion

The galactic disc inflation by a growing magnetic field may be important for the generation of magnetic field at early stages of galactic evolution. It was claimed recently that large-scale magnetic fields of strength comparable to their nowadays value can be present in very young galactic discs at redshifts of about 2 [21, 22]; such a field has been definitely detected in a galaxy at the redshift ≈ 0.395 [23]. It has been suggested in ref. 24 that this can be explained by a relatively strong initial field in the interstellar gas. Our results allow to suggest a further refinement of the argument: a fast generation of magnetic field at an early stage of galactic evolution may be facilitated by super-exponential field amplification due to the inflation of the disc by the magnetic field. This possibility should be thoroughly studied using detailed models of galactic evolution.

In our Galaxy, reversals of the magnetic field along the radius are observed [25, 26]. These reversals plausibly represent nonlinear transient structures that are more likely to persist when the dynamo action is stronger [14]. Hence, an enhancement of the dynamo owing to the dependence of the disc thickness on themagnetic field may be of importance for the evolution of the reversals.

In a number of galactic dynamo models, effects associated with the time variation of the disc thickness have been explored [27, 28, 29, 30] basing on the dependence of h on the *turbulent* pressure ϱv^2 as the most important time-dependent pressure component. It was supposed that v^2 increases when star formation is enhanced. However, we believe that the above mechanism, i.e., the disc inflation by *magnetic* pressure, is much more important.

The magnetic part of the pressure grows due to the dynamo action. The thermal and turbulent parts of the pressure depend on the turbulent velocity whose amplitude is connected with the intensity of star formation but is bounded from above by the sound speed. In other words, the upper bound for the turbulent velocity is determined by the temperature of the interstellar gas. We emphasize that the turbulent velocity observed in spiral galaxies is quite close to this limit ($v \approx v_\mathrm{s} \approx 10\,\mathrm{km\,s^{-1}}$ in the warm interstellar gas) [31].

Several gas phases are represented in the interstellar medium, which have different temperatures, densities and filling factors. The large-scale magnetic field is generated probably only in one of them, namely in the so-called *warm* phase [3], which is now believed to occupy a major part of the interstellar space. The temperature of the warm interstellar gas is approximately $10^4\,\mathrm{K}$ [31] and hardly depends on the rate of star formation or similar parameters. That is why the turbulent pressure in interstellar medium cannot increase substantially due to the bursts of star formation, as suggested in Refs. 27, 28, 29 and 30. Galaxy formation models [32] confirm that, soon after the formation of the gaseous disc, radiative losses keep the gas temperature at the level $10^4\,\mathrm{K}$, i.e. $v \le 10\,\mathrm{km\,s^{-1}}$ during most of the galactic lifetime.

It is also important to note that the duration of star formation bursts in spiral galaxies does not exceed $10^8\,\mathrm{yr}$, which is significantly shorter than the time scale of the large-scale magnetic field amplification; this fact also alleviates the direct effect of star formation bursts on the dynamo.

The energy ejected into the interstellar matter during a star formation burst is spent to generate and maintain the *hot* phase of the interstellar gas ($T \simeq 10^6\,\mathrm{K}$) which plausibly occupies only a small fraction of the total volume [33, 34]. The hot phase of the interstellar gas is unsteady: bubbles of hot gas rise from the disc to fill the gaseous halo of the galaxy and thus the energy balance in the disc is maintained without inflating the layer of the warm gas. The typical time in which the hot gas leaves the disc is of the order $10^7 - 10^8$ yr [33], which is much shorter than the dynamo time scale, $\tau \simeq 10^9$ yr. Hence the hot interstellar gas hardly affects *directly* the generation of the large-scale magnetic field in spiral galaxies.

We conclude that the time variation of the thickness of the *warm* interstellar gas layer, where the large-scale magnetic field is generated, is due mainly to the time variation of the field itself and not to variations in turbulent pressure.

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Figure Captions

Fig. 1 The local growth rate of the large-scale magnetic field in the Solar vicinity of the Milky Way as a function of the half-thickness of the disc according to a numerical solution of Eqs. (12) (solid). Dotted lines illustrate the asymptotic forms (4) (with $C_1 \approx 1.4$) and (5) (with $C_2 \approx 0.5$) (when evaluating the asymptotics, the critical dynamo number was adopted as $D_{\rm cr} \approx 7$ to comply to the numerical solutions).

Fig. 2 The time variation of the dynamo time scale for different values of B_h/B_α for the Solar vicinity of the Milky Way, $r_\odot = 10\,{\rm kpc}$. The dependence of h on B adopted is given in Eq. (6). Dashed: $B_h/B_\alpha = 10$; thin solid line: $B_h/B_\alpha = 1$; thick solid line: $B_h/B_\alpha = 0.5$.

Fig. 3 The same as in Fig. 3 but for the dependence of h on B as given in Eq. (8).