ANALYTICAL STUDIES OF MHD OUTFLOWS

Self-similar solutions

Outflows from stellar bodies

From Solar Wind
To Stellar winds (O M stars)
Compact objects (pulsars, …)
AGN
GRB

From 1-D Hydro steady model (Parker 1958)
To 3-D MHD relativistic time dep. codes

Collimated flows: **Steady** and **axisymmetric**
MHD approach

Steadiness and axial symmetry ($\partial/\partial t = \partial/\partial\phi = 0$)

$$[\mathbf{B}\,,\ \mathbf{V}] = [\mathbf{B}_p, \mathbf{V}_p] + [B_\phi\,,\ V_\phi]\mathbf{e}_\phi$$

- $\mathbf{B}_p$ and $\rho\mathbf{V}_p$ expressed through the functions $\Psi(r,\theta)$ and $A(r,\theta)$ ($\Psi_A = \mathrm{d}\Psi/\mathrm{d}A$):

$$\mathbf{B}_p = \nabla \times \left[\frac{A}{r\sin\theta}\mathbf{e}_\phi\right], \qquad \rho\mathbf{V}_p = \nabla \times \left[\frac{\Psi}{r\sin\theta}\mathbf{e}_\phi\right]$$

$$\mathbf{V}_p = \frac{\Psi_A}{4\pi\rho}\mathbf{B}_p\,, \qquad M^2 = \frac{V_p^2}{V_{a,p}^2} \equiv \frac{\Psi_A^2}{4\pi\rho}$$

- B_ϕ, V_ϕ expressed in integral form:

$$B_\phi \propto \frac{1 - [\Omega\, r^2 \sin^2\theta]/L}{1 - M^2}$$

$$V_\phi \propto \frac{[\Omega\, r^2 \sin^2\theta]/L - M^2}{1 - M^2}$$

- $L(A) \propto$ *total angular momentum*
- $\Omega(A) \propto$ *angular velocity of the fieldlines*
- $r^2\sin^2\theta|_{M=1} = L/\Omega$

Bernoulli + Transfield $\longrightarrow$

$$(1-M^2)\left(a_1\frac{\partial^2 A}{\partial r^2}+\frac{2a_2}{r}\frac{\partial^2 A}{\partial r\,\partial\theta}+\frac{a_3}{r^2\sin^2\theta}\frac{\partial^2 A}{\partial\theta^2}+a_4\right)$$

$$+a_5=0$$

where a_i are complicated functions
The nature of this Eq. ruled by:

$$D=a_2^2-a_1\,a_3=$$

$$=(V_p^2-V_f^2)(V_p^2-V_s^2)(V_p^2-V_c^2)(V_a^2+C_s^2)$$

- $D<0$: *elliptic*, Dirichlet B.C. on a *closed* surface
- $D>0$: *hyperbolic*, Cauchy B.C. on an *open* surface

• Our Eq. is of *mixed* type: in different regions we can have $D>0$ or $D<0$

• Singularities for $M^2=1$ and $V_p=V_s$, V_f

•• *Relation between the singularities and the hyperbolic/elliptic regions ?*

Singularities and hyperbolic/elliptic regimes

Slope of the characteristics in the hyp. regions

$$r\frac{\mathrm{d}\theta}{\mathrm{d}r} = \frac{1}{a_1}(a_2 \pm \sqrt{D})$$

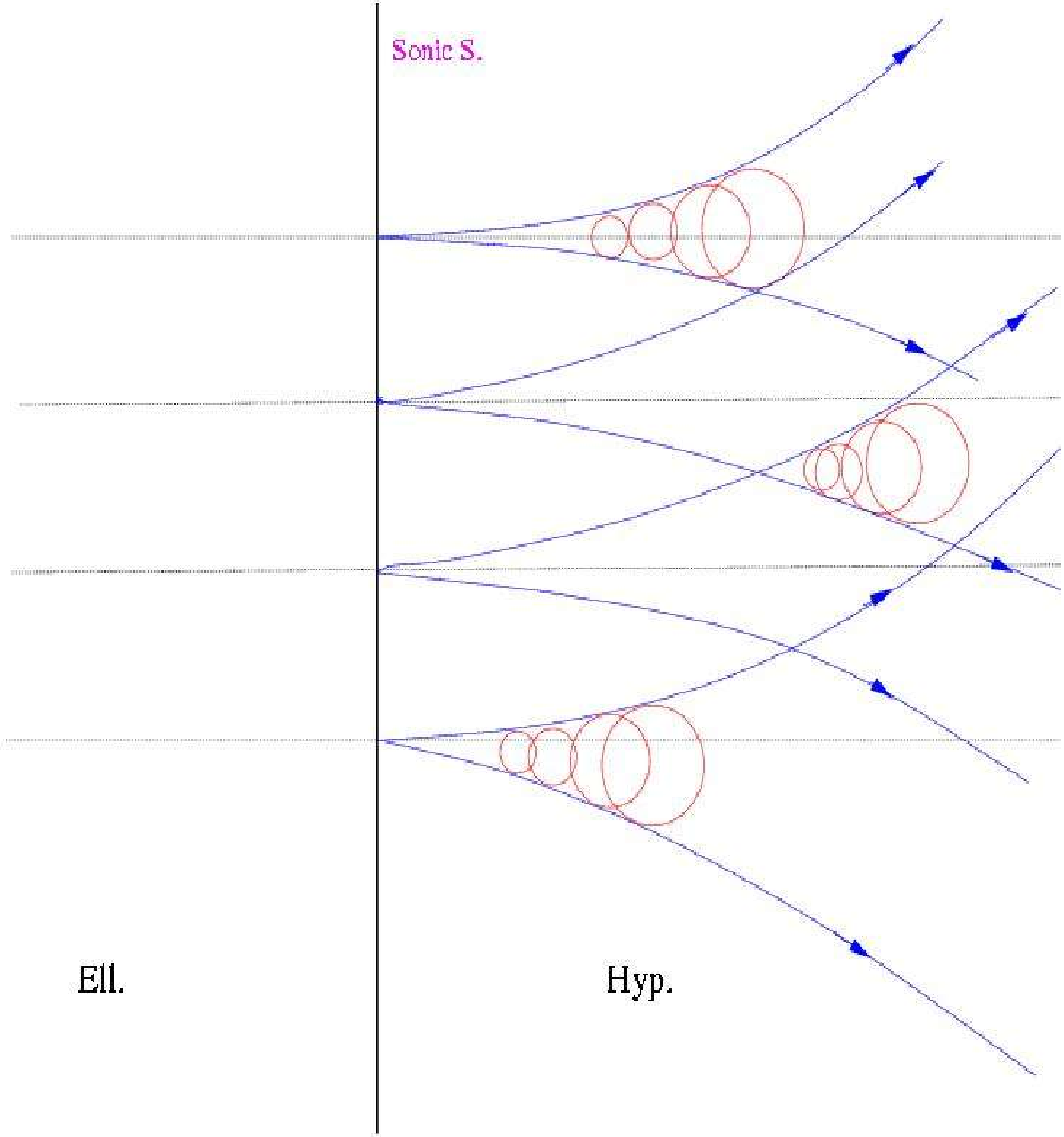
For $a_1 = 0$ one of the characteristics direction becomes parallel to $\mathbf{e}_\theta$:

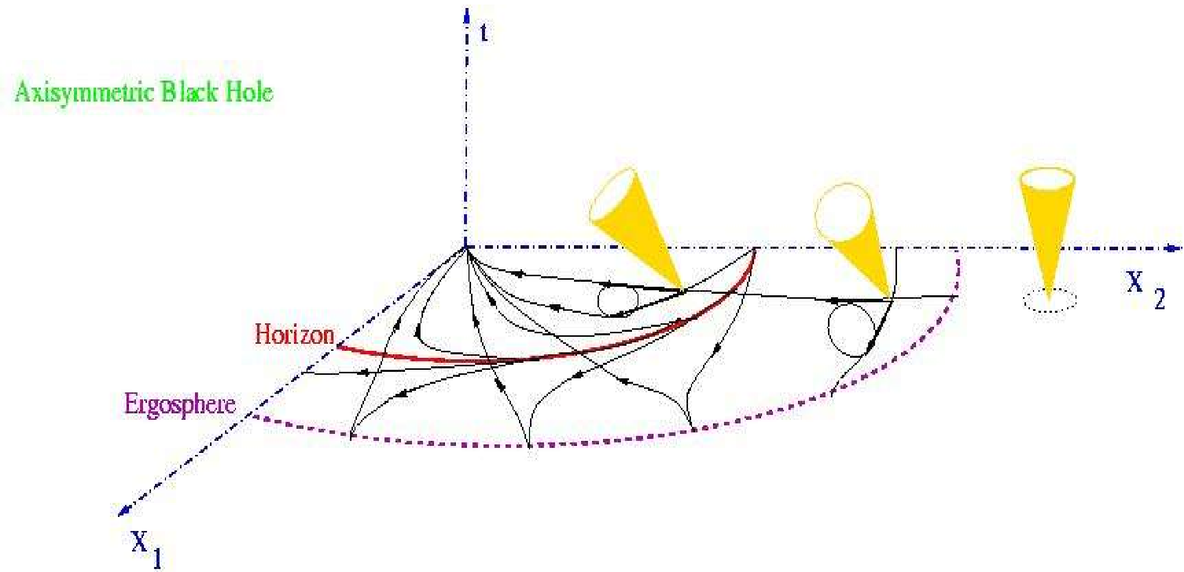
Limiting Characteristics or *Separatrix*

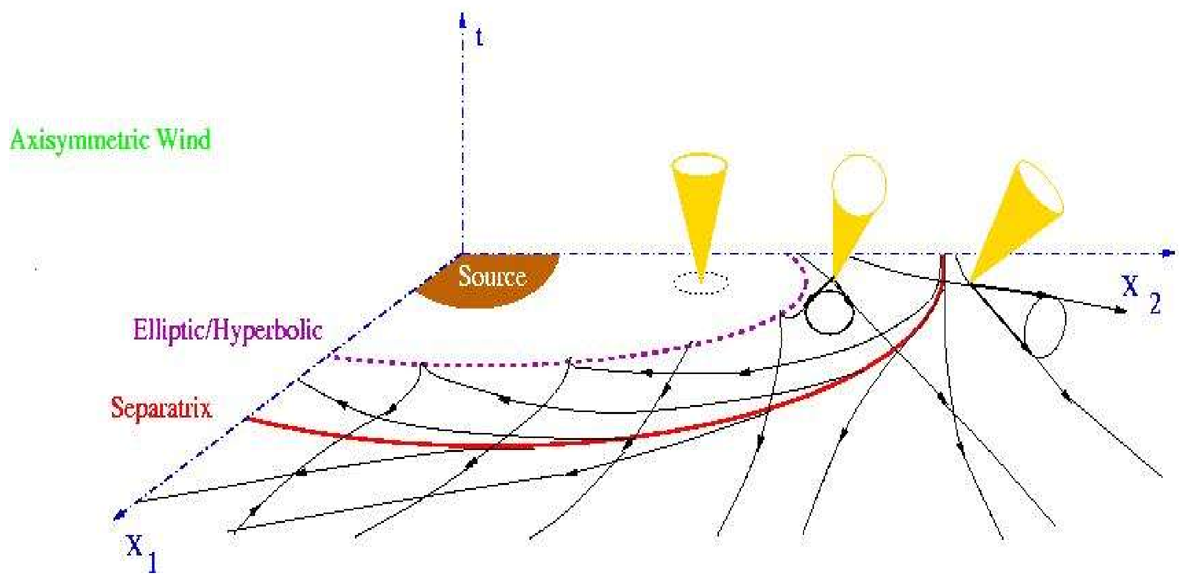
- If the surface of the *L.C.* is not $\perp$ to $\mathbf{V}_p$ it *does not coincide* with the positions where $D = 0$
- The *L.C.* defines the regions *causally connected* inside the outflow

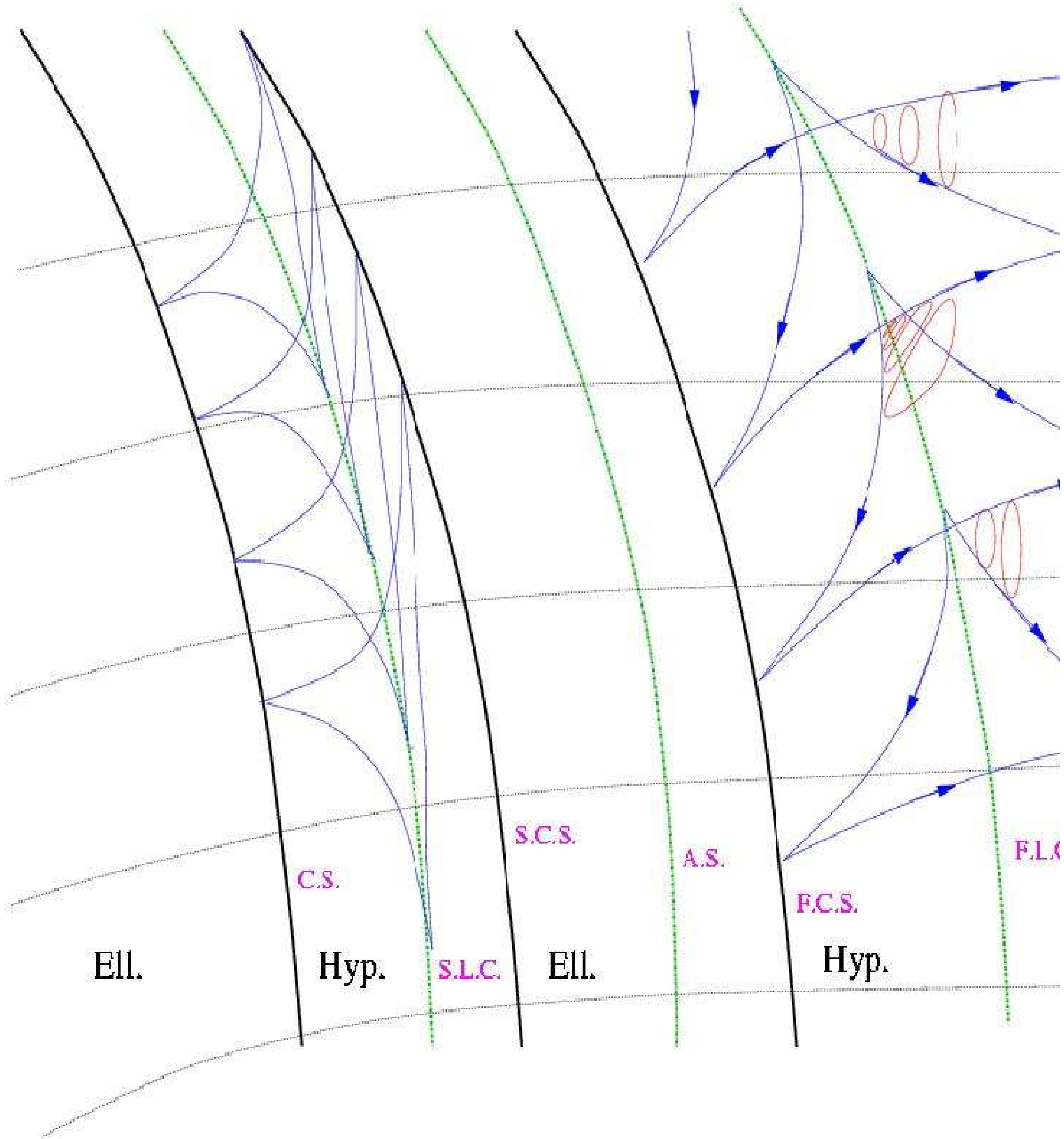
⋆ Classical hydrodynamical problem in 2-D supersonic flows (Landau & Lifshitz 1959)

To construct a correct solution we need to know the *L.C.*, *but this would require to know this solution !*


Sonic S.
Ell.
Hyp.


t
Axisymmetric Black Hole
Horizon
Ergosphere
X 2
X 1


t
Axisymmetric Wind
Source
Elliptic/Hyperbolic
Separatrix
X 2
X 1



S.C.S.
A.S.
C.S.
F.C.S.
Ell.
Hyp.
S.L.C.
Ell.
Hyp.

Self-similarity

- *Separation of the variables*: $A(r,\theta) \propto f(r)\, g(\theta)$
- *Scaling of one of the variables with one coordinate*

Partial $\rightarrow$ **Ordinary** diff. eqs.:

$$a_1 g(\theta) \frac{\mathrm{d}^2 f(r)}{\mathrm{d}r^2} + \frac{2a_2}{r} \frac{\mathrm{d}f(r)}{\mathrm{d}r} \frac{\mathrm{d}g(\theta)}{\mathrm{d}\theta}$$

$$+ \frac{a_3}{r^2 \sin^2\theta} f(r) \frac{\mathrm{d}^2 g(\theta)}{\mathrm{d}\theta^2} + a_4 + \frac{a_5}{1 - M^2} = 0$$

Radial: $f(r) \propto r^x$, $\quad M(\theta)$

Singularities (L.C.) for $a_3 = 0$, where $V_\theta = V_{s,\theta}, V_{f,\theta}$

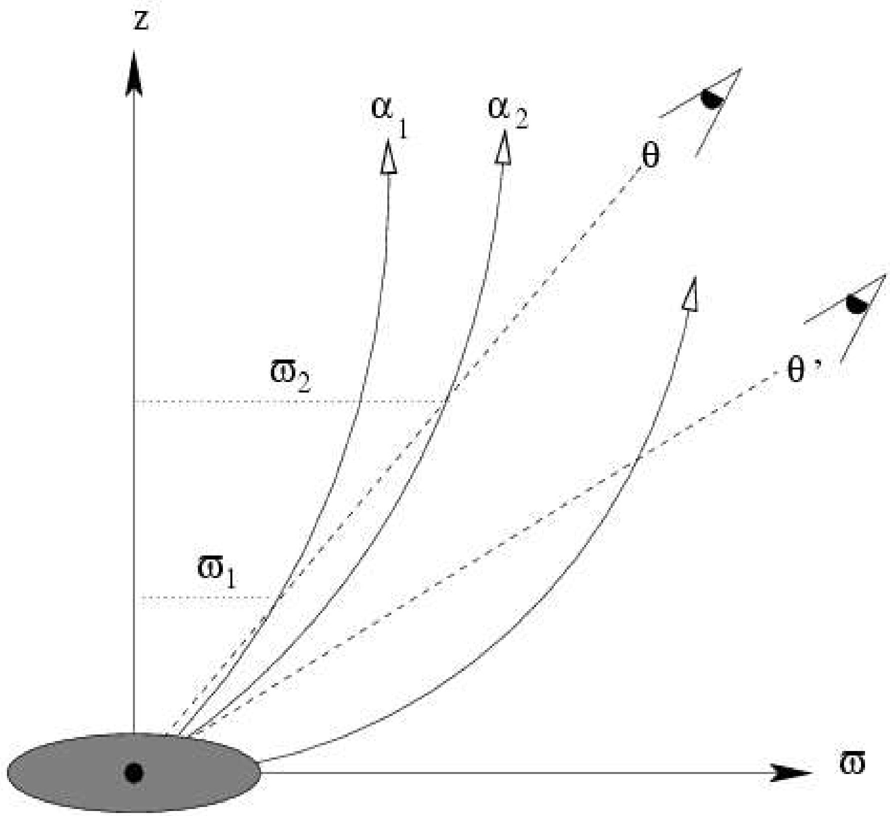
Magneto - Centrifugal Disk winds

Merid.: $g(\theta) \propto \sin^2\theta$, $\quad M(r)$

Singularities (L.C.) for $a_1 = 0$, where $V_r = V_{s,r}, V_{f,r}$

Magneto - Thermal winds

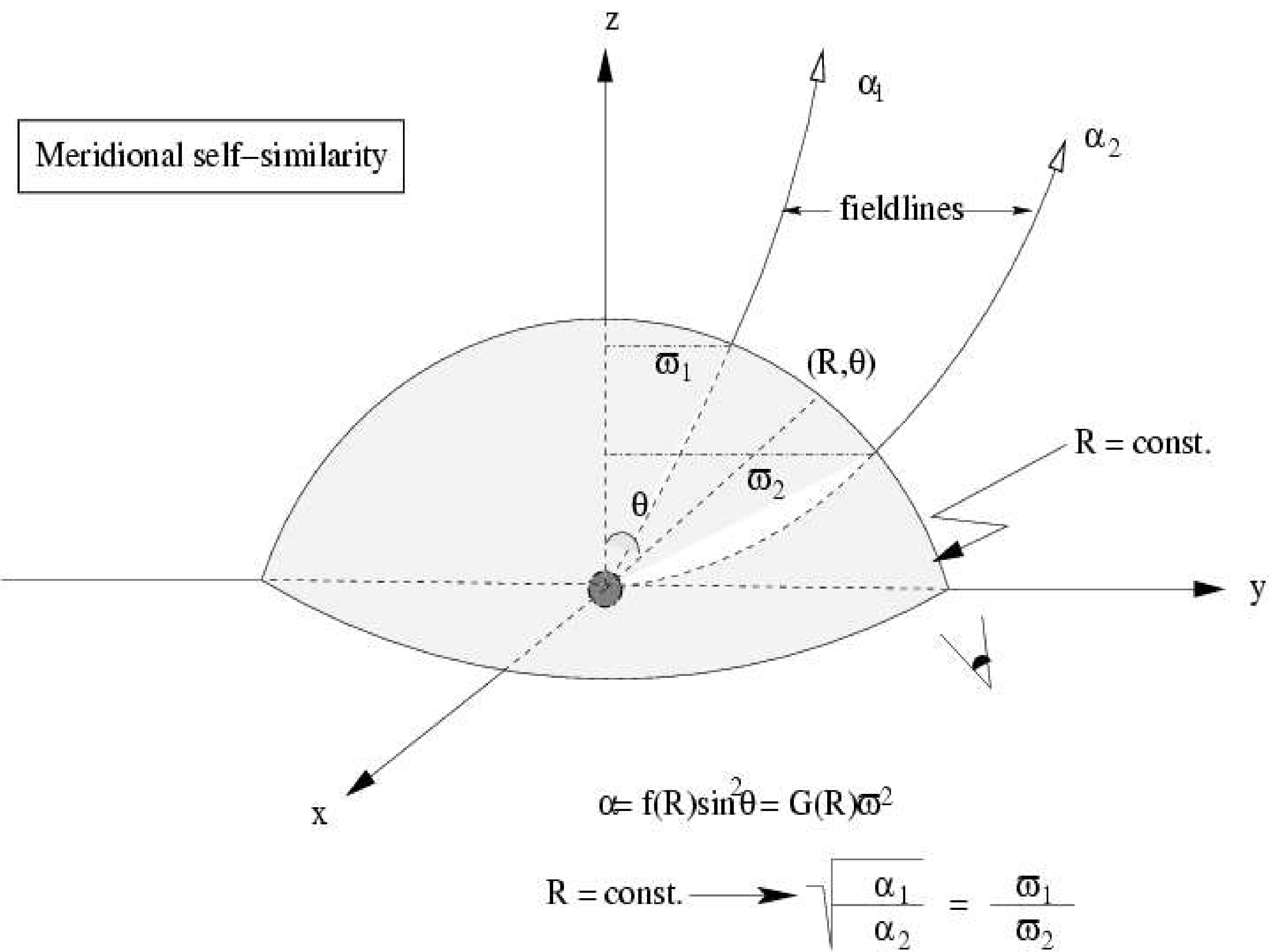
Radial self–similarity



$$\alpha = g(\theta) R^{x} = F(\theta)\,\varpi^{x}$$

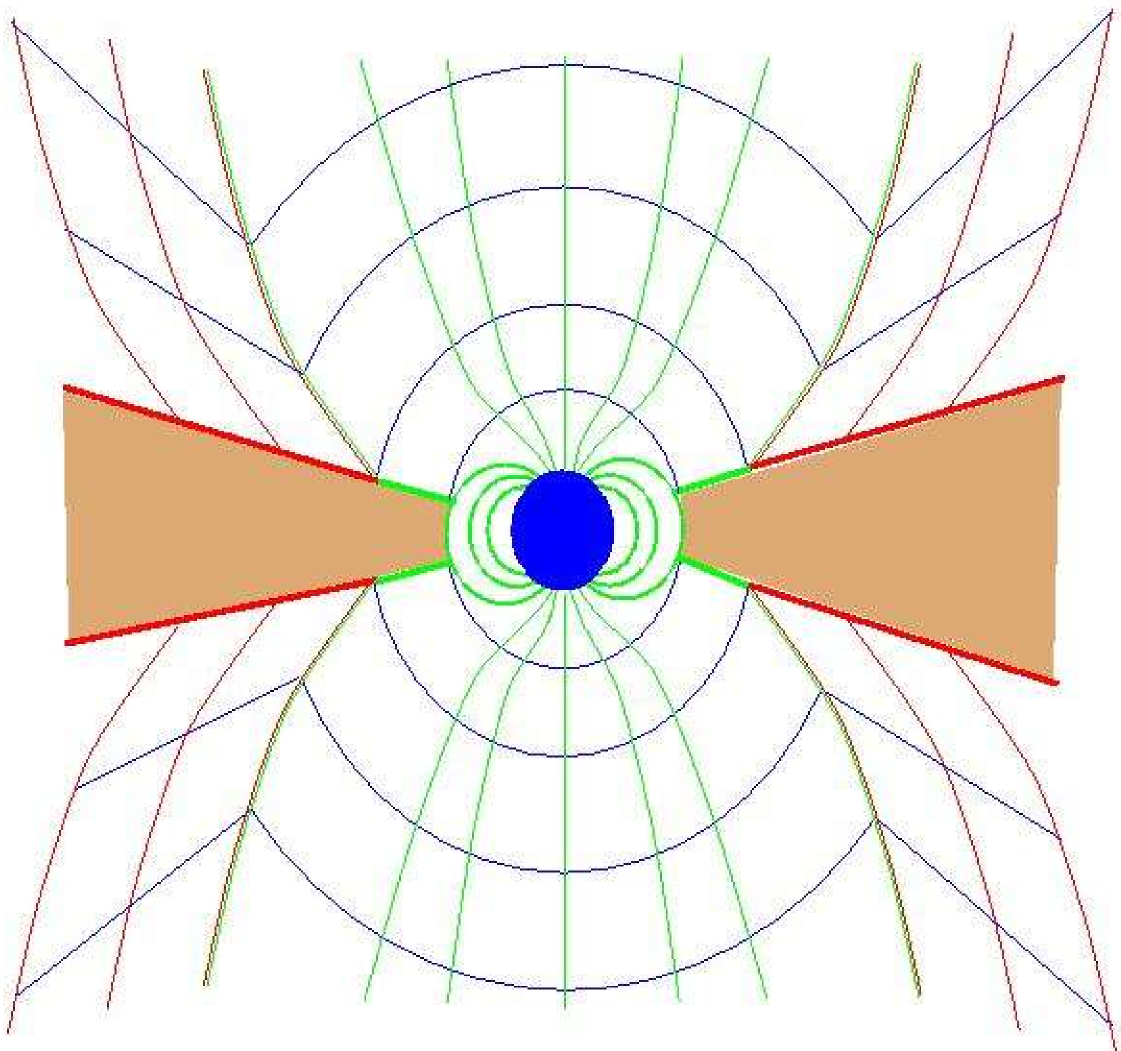
$$\theta = \text{const.} \longrightarrow \frac{\varpi_1}{\varpi_2} = \left(\frac{\alpha_1}{\alpha_2}\right)^{1/x}$$

Blandford & Payne ‘82, ……



Low & Tsinganos ‘85,   Sauty et al. ’94, …….

SELF-SIMILAR JETS



Meridional self-similar models

(Meliani, Sauty, Tsinganos, Trussoni, Vlahakis)

$M(R) \rightarrow$ Surfaces of constant M: spheres

$I_z \propto L\Psi_A \propto \lambda\alpha$

$P(R,\alpha) \propto P_o(R)(1+\kappa\alpha)$

$\Psi_A^2(\alpha) \propto 1+\delta\alpha$

From numerical integration we have $\mathcal{F}(R;\lambda,\kappa,\delta,....)$

The solutions are suitable to model outflows in the region around the rotational axis

Acceleration: basically thermal

Collimation: thermal or magnetic

The collimation depends on the interplay between the inwards and outwards forces across the streamlines

The thermal or magnetic confinement is ruled by the following parameter

$$\epsilon \propto E_P - E_c + \Delta E_G$$

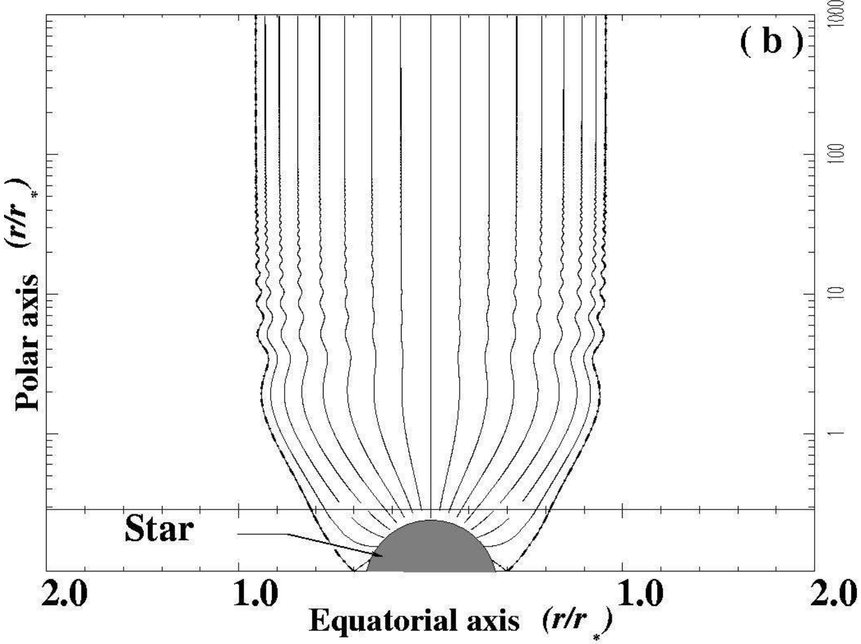
with

E_P: Variation of the Poynting energy

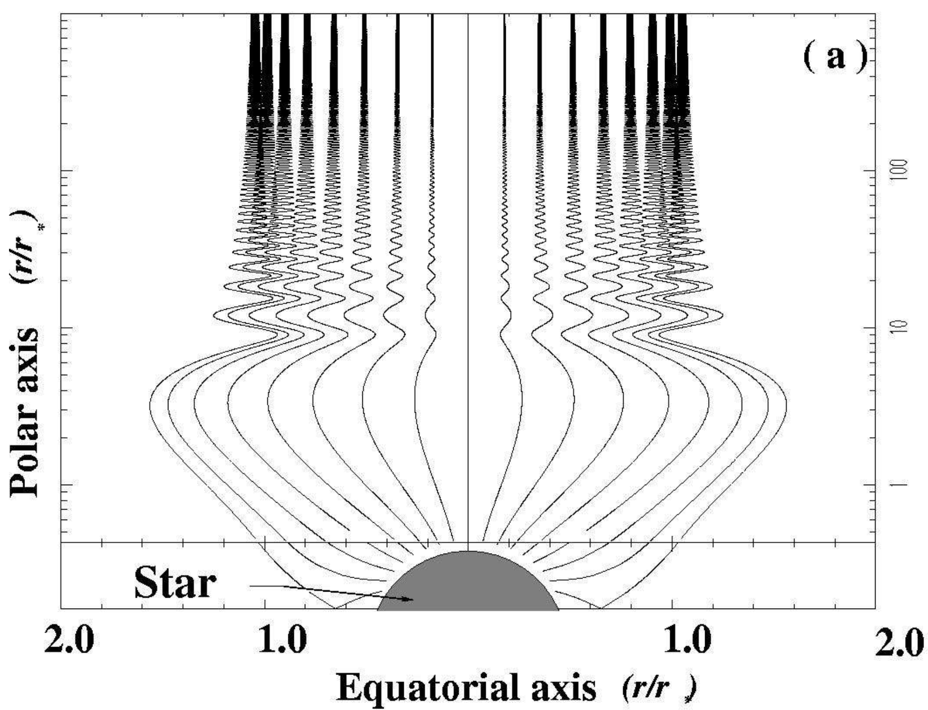
E_c: Variation of the centrifugal energy

$\Delta E_G \propto (\kappa - \delta)$: Variation of the gravitational energy

- $\epsilon > 0$ *Efficient magnetic rotator*: magn. collimation
- $\epsilon < 0$ *Inefficient magn. rotator*: thermal. coll.



$\varepsilon > 0$

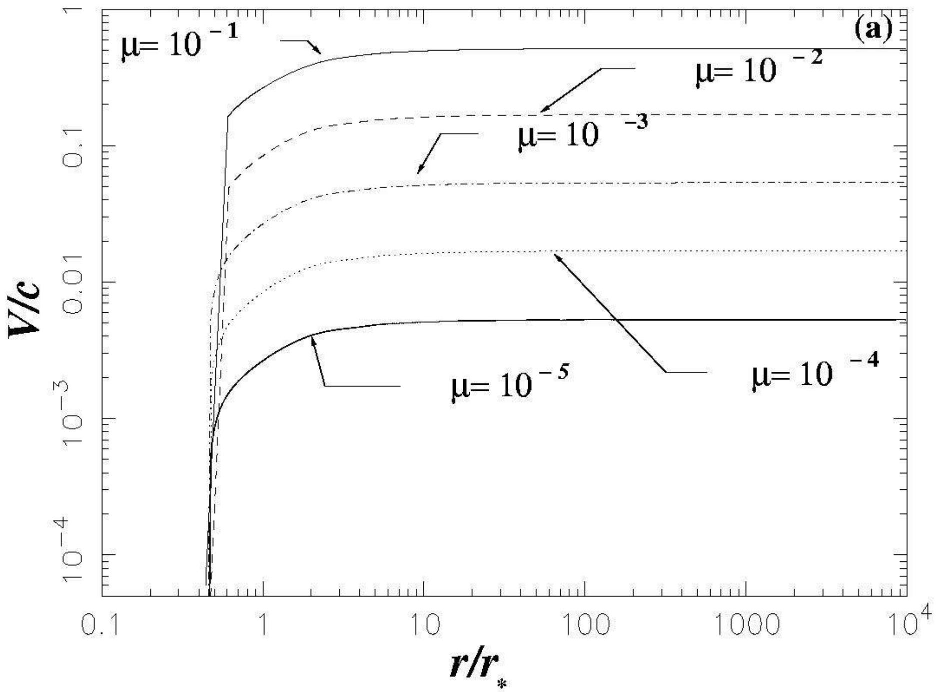


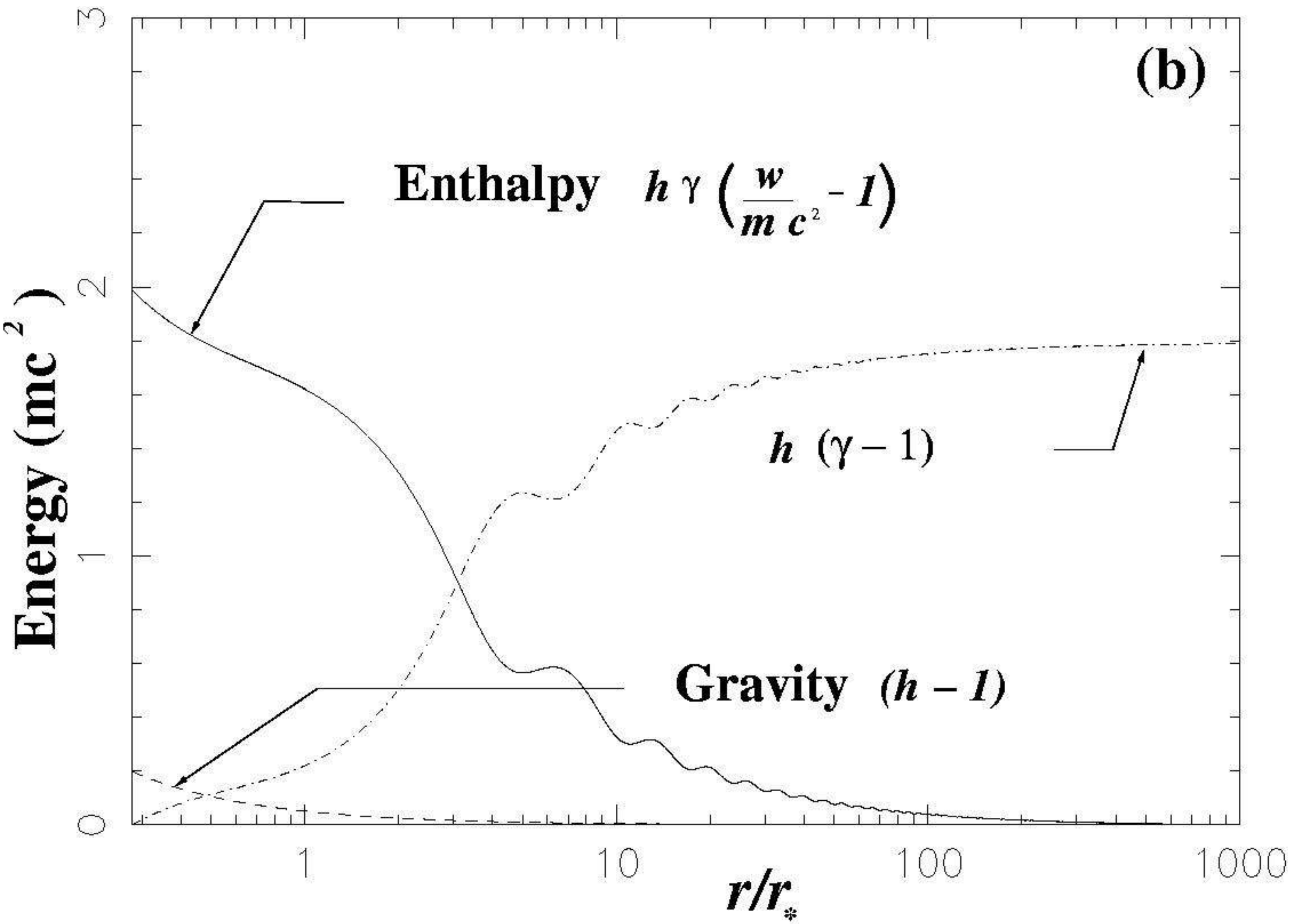
$\varepsilon < 0$

Relativistic Extension

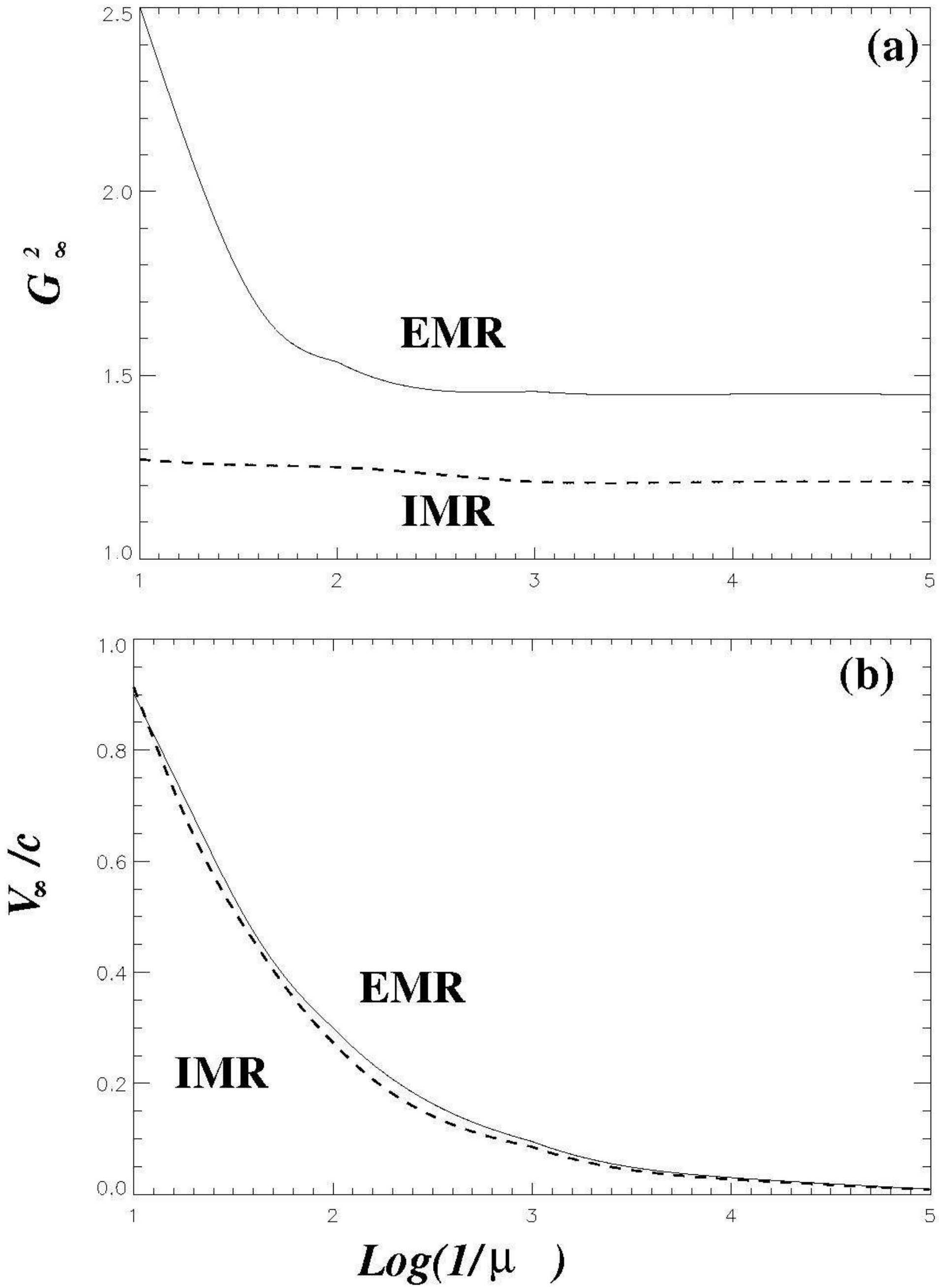
- New parameter: $\mu = r_g / r_\star < 1$
- Non relativistic rotational velocity: $\varpi \ll c/\Omega$

- *Mildly relat. velocities*
- *Electric force* [$\rho \mathbf{E} \propto -\rho\,(\mathbf{V} \times \mathbf{B}\,)\,/\,\mathrm{c}$]: *decoll.*



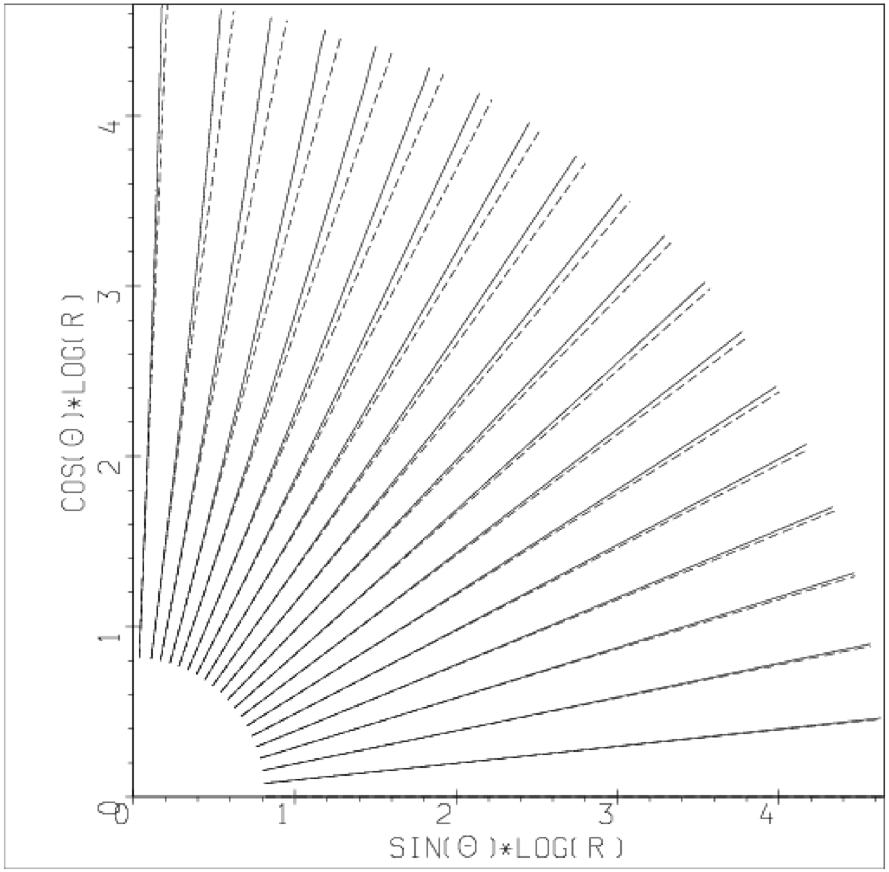


(b)
Enthalpy $h\gamma\left(\frac{w}{mc^2}-1\right)$
$h\ (\gamma-1)$
Gravity $(h-1)$
Energy (mc ^{2})
r/r_*
0
1
2
3
1
10
100
1000



(a)
2.5
2.0
1.5
1.0
G 2 ∞
EMR
IMR
1
2
3
4
5
(b)
1.0
0.8
0.6
0.4
0.2
0.0
V∞ /c
EMR
IMR
1
2
3
4
5
Log(1/μ)

Weak collim. in relat. outflows
time dependent computations
(Tsinganos & Bogovalov,
2002-2005)



Possible Model:

Inner relativistic wind
Outer classical disk wind

CONCLUSIONS

Through the analysis of MHD steady solutions we can have insights on the physical properties of astrophysical outflows

Further issues:

Non ideal MHD (resistivity)
Radiative effects (losses and/or acceleration)
Large amplitute e.m. waves

Time dependent solutions: Asympt. status, instabilities
Interaction with the envir.
Launch

…….