3-D GRMHD simulations of Disk-Jet Coupling and Associated Variabilities and Emission

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ULTRA-RELATIVIATIC JETS IN ASTROPHYSICS, **Observations, Theory, Simulations**

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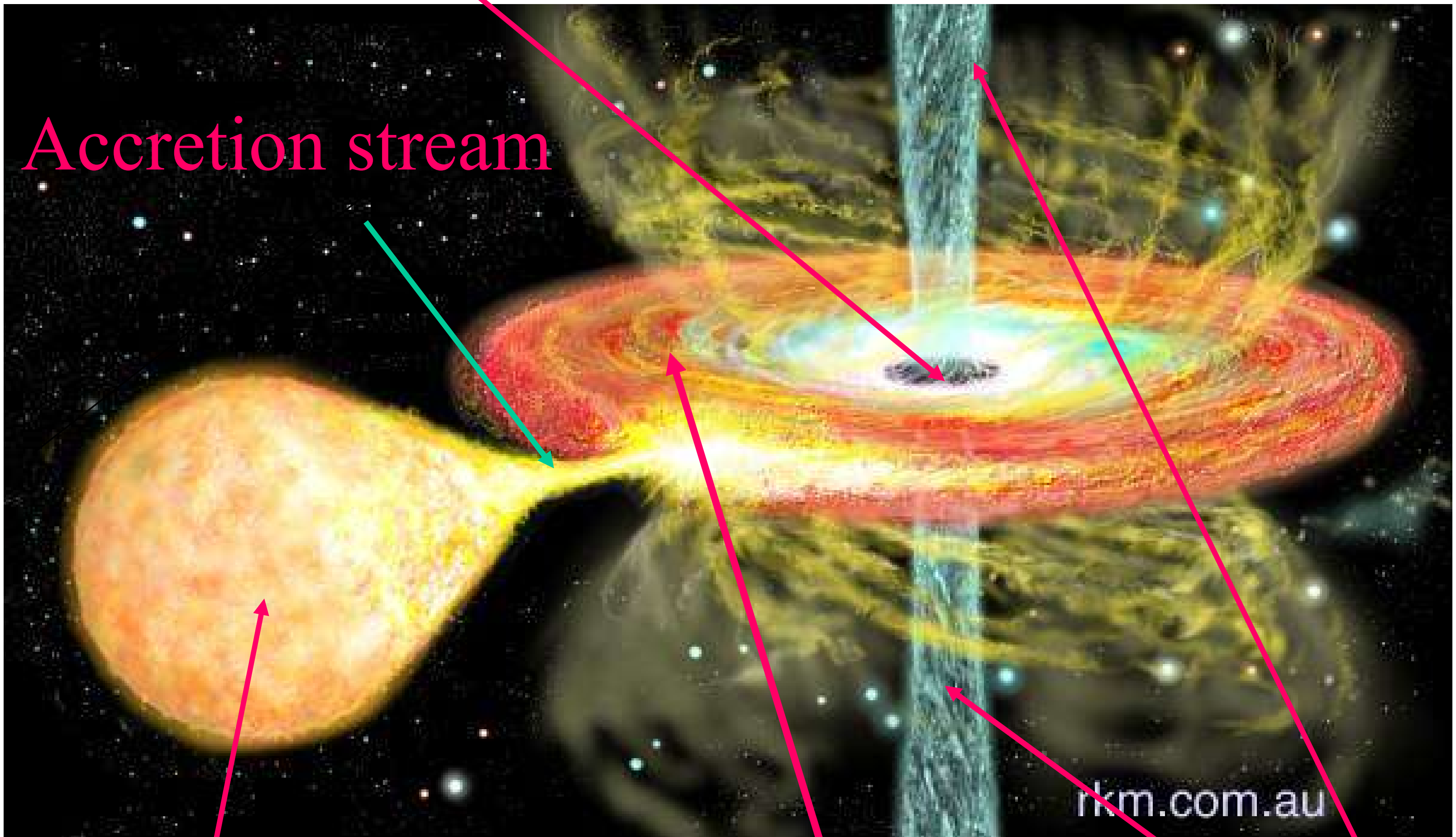
- Scientific objectives
- Initial conditions
- Recent 3-D GRMHD simulations of jet formation
- Comparisons with simulations using different initial magnetic fields
- Black body radiation from Kerr black hole using relativistic ray-tracing
- Future plans

Scientific objectives

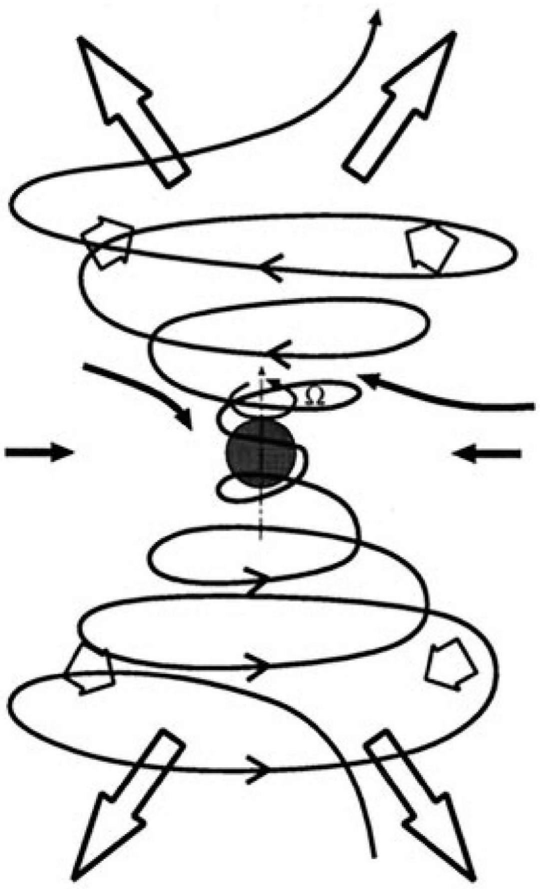
- **How do accretion disks near black holes evolve under the influence of accretion-ejection instabilities?**
- **How do these instabilities affect states of black hole vicinity and associated jet formation?**
- **How do our full 3-D GRMHD simulations with a Kerr black hole support Blandford-Znajek model?**
- **What is the main mechanism of relativistic jet formation?**
- **How is the relativistic jet collimated in the process of its formation?**
- **How do relativistic jets propagate with perturbations?**

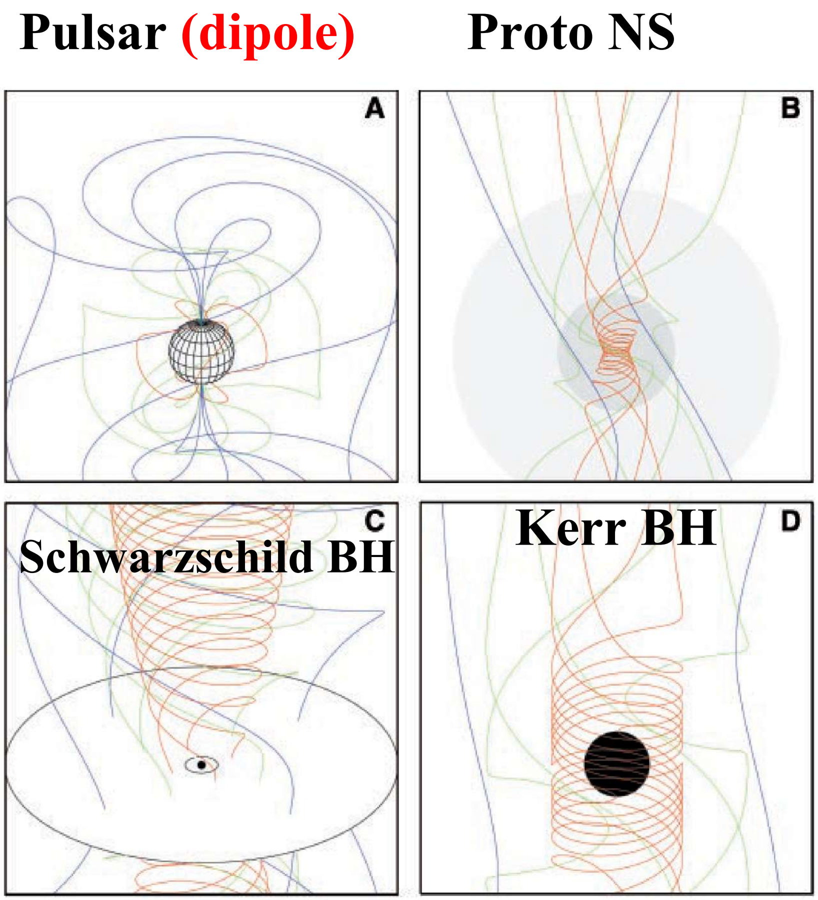
Jets from binary stars

(Schematic figure)



Schematic mechanism for accelerating and collimating jets



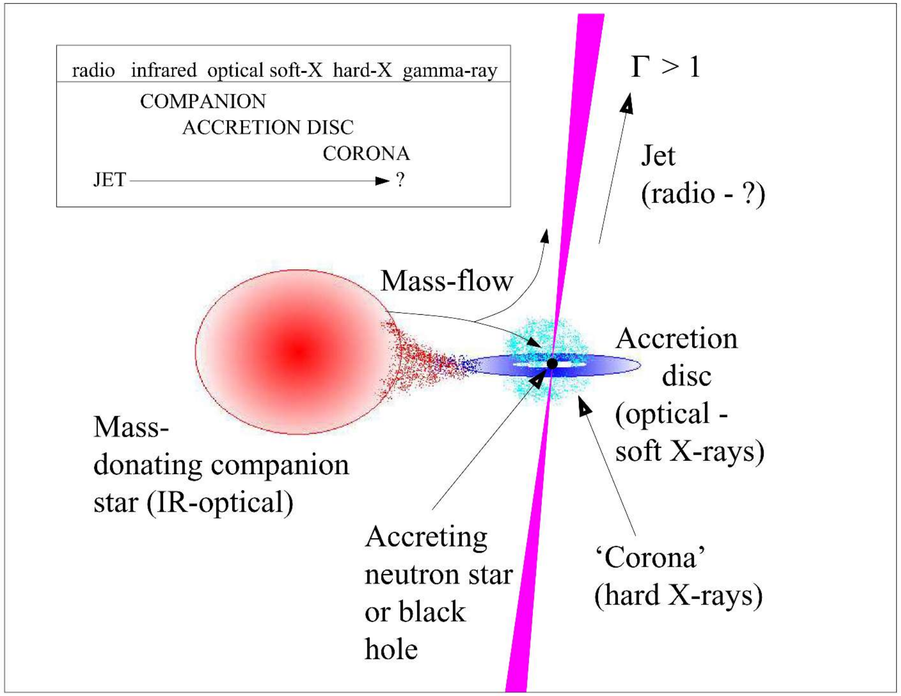


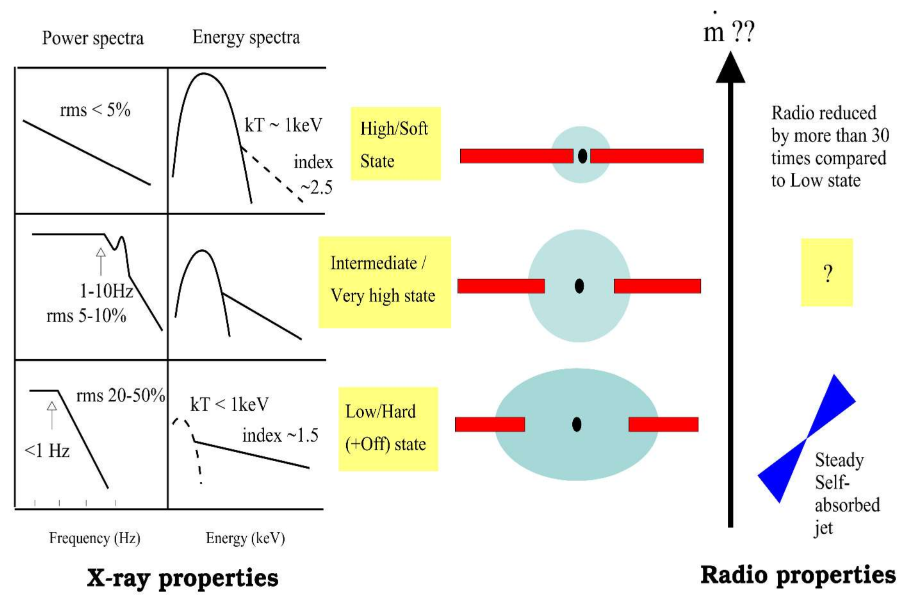
Meier et al. Science, 2001

X-ray binary system

States of BHCs

(Fender 2001)





- ***How does the mass accretion rate determine the state?***
- ***How does the angular momentum, j affect the state?***
- ***How is instability involved with the state transition?***

3. Simulation models

- General relativistic MHD codes
  axisymmetric (2-D) and full 3-D models
  Schwarzschild and Kerr black holes
  with simplified Total Variation Diminishing (TVD) method (Davis 1984)
  (Lax-Wendroff's method with additonal diffusion term)

Tortoise Coordinates

$d/dr_* \equiv (r - r_S) d/dr$ $\qquad$ $r_* = \ln(r - r_S)$

Schwarzschild radius: $r_S \equiv 2GM_{BH}/c^2$

Time Constant: $\tau_S \equiv r_S/c$

Boundary conditions at $r = 1.1\ r_S,\ 20\ r_S$: radiating

CFL numerical stability condition is severe at $r = 1.5\ r_S$

Polytropic equation of state: $p = \rho^\Gamma$

$\Gamma = 5/3$ and $H = 1.3$

Initial conditions

•Free-falling corona (these simulations)

accretion disk

relativistic Keplerian velocity $v_\theta = v_K \equiv c/[2(r/r_S - 1)]^{1/2}$

$\rho = \rho_{ffc} + \rho_{dis}$ $\qquad r_D \equiv 3\, r_S$

$$\rho_{dis} = \begin{cases} 100\,\rho_{ffc} & \text{if } r > r_D \text{ and } |\cot\theta| < \delta \\ 0 & \text{if } r \le r_D \text{ or } |\cot\theta| \ge \delta \end{cases}$$

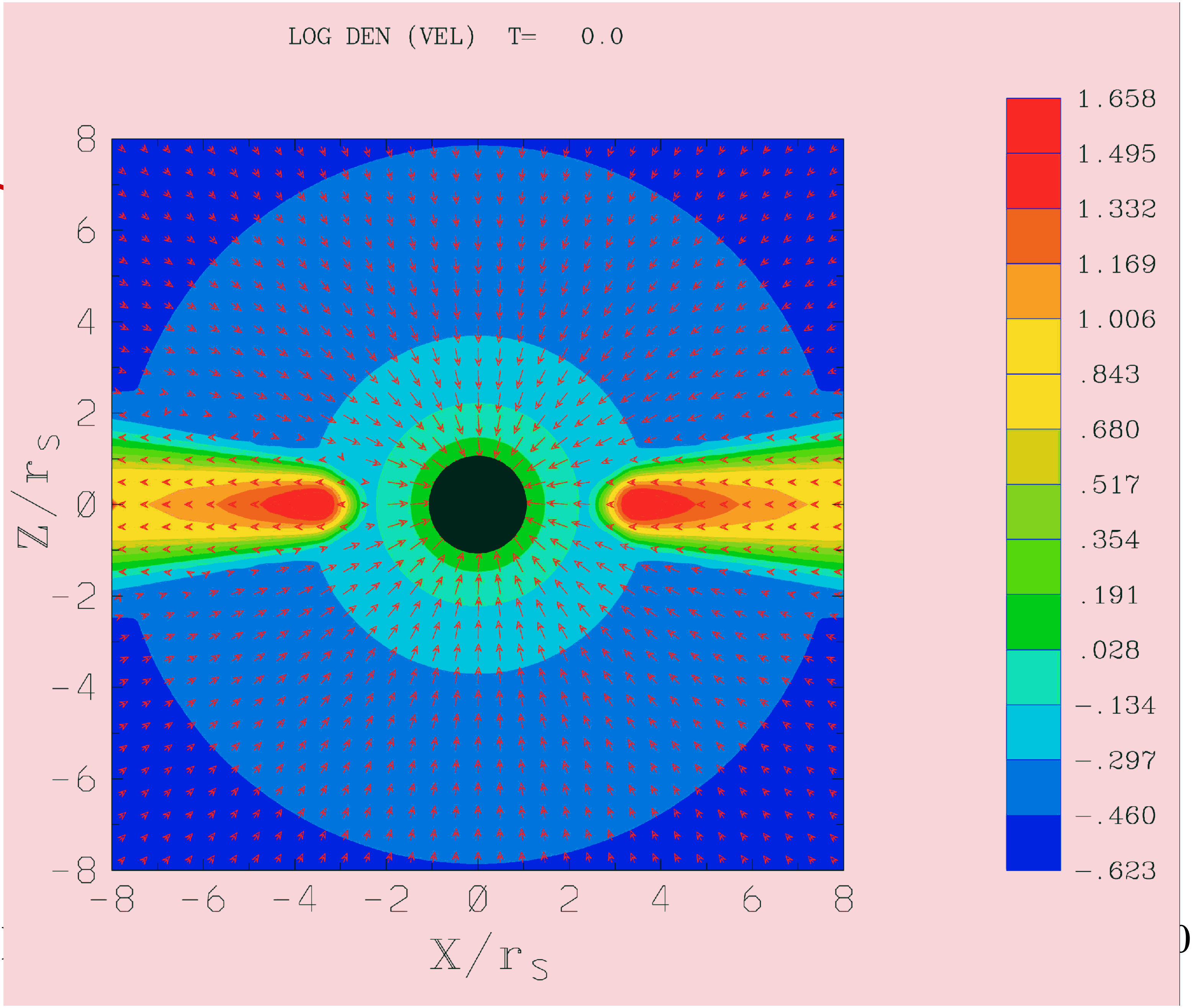
($\delta = 0.125$)

δ: thickness of disk

$$(v_r, v_\theta, v_\varphi) = \begin{cases} (0, 0, v_K) & \text{if } r > r_D \text{ and } |\cot\theta| < \delta \\ (-v_{ffc}, 0, 0) & \text{if } r \le r_D \text{ or } |\cot\theta| \ge \delta \end{cases}$$

3-D simulation mass density ($\log_{10} \rho$)

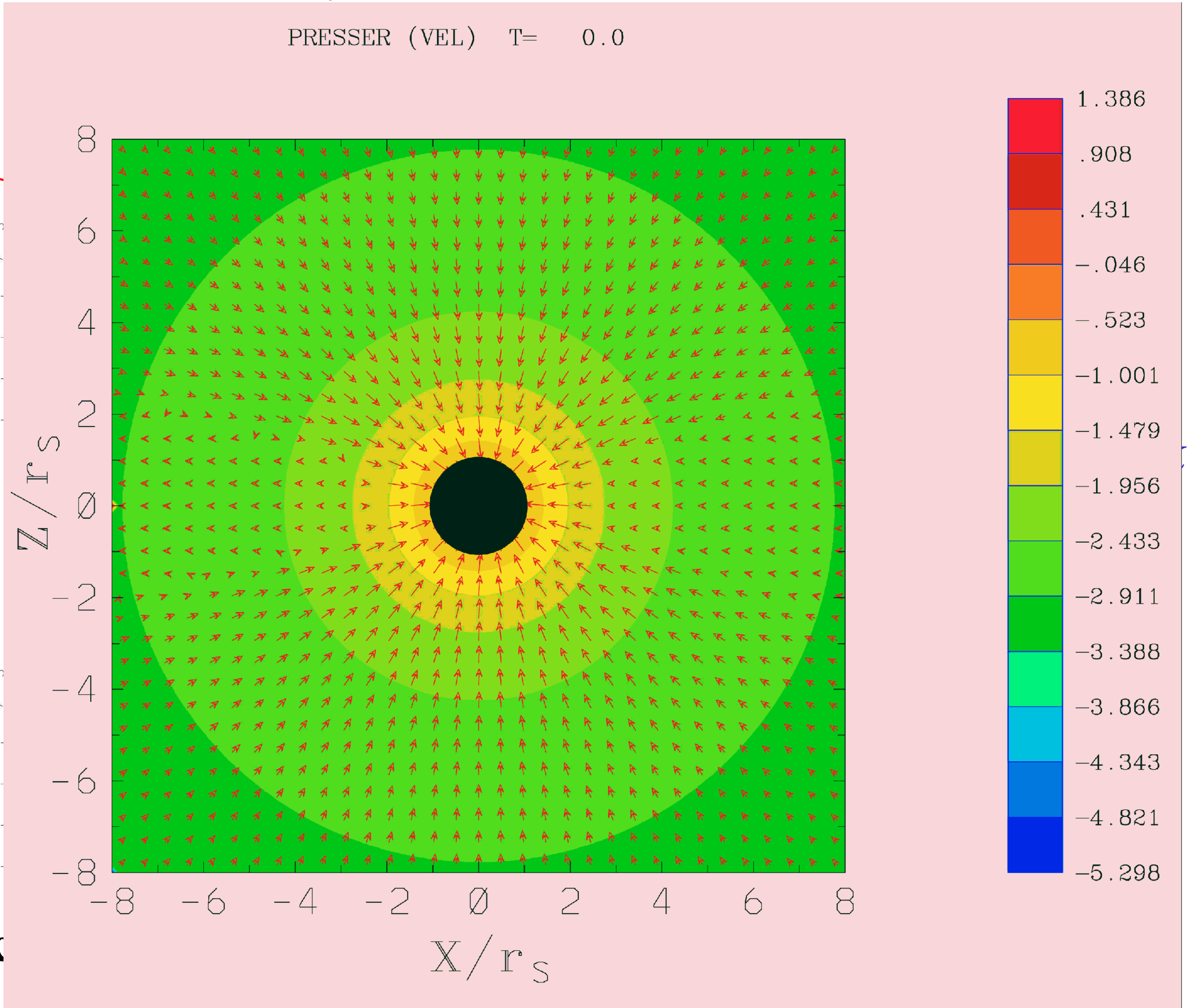
Accretion



● Blac ... 05)

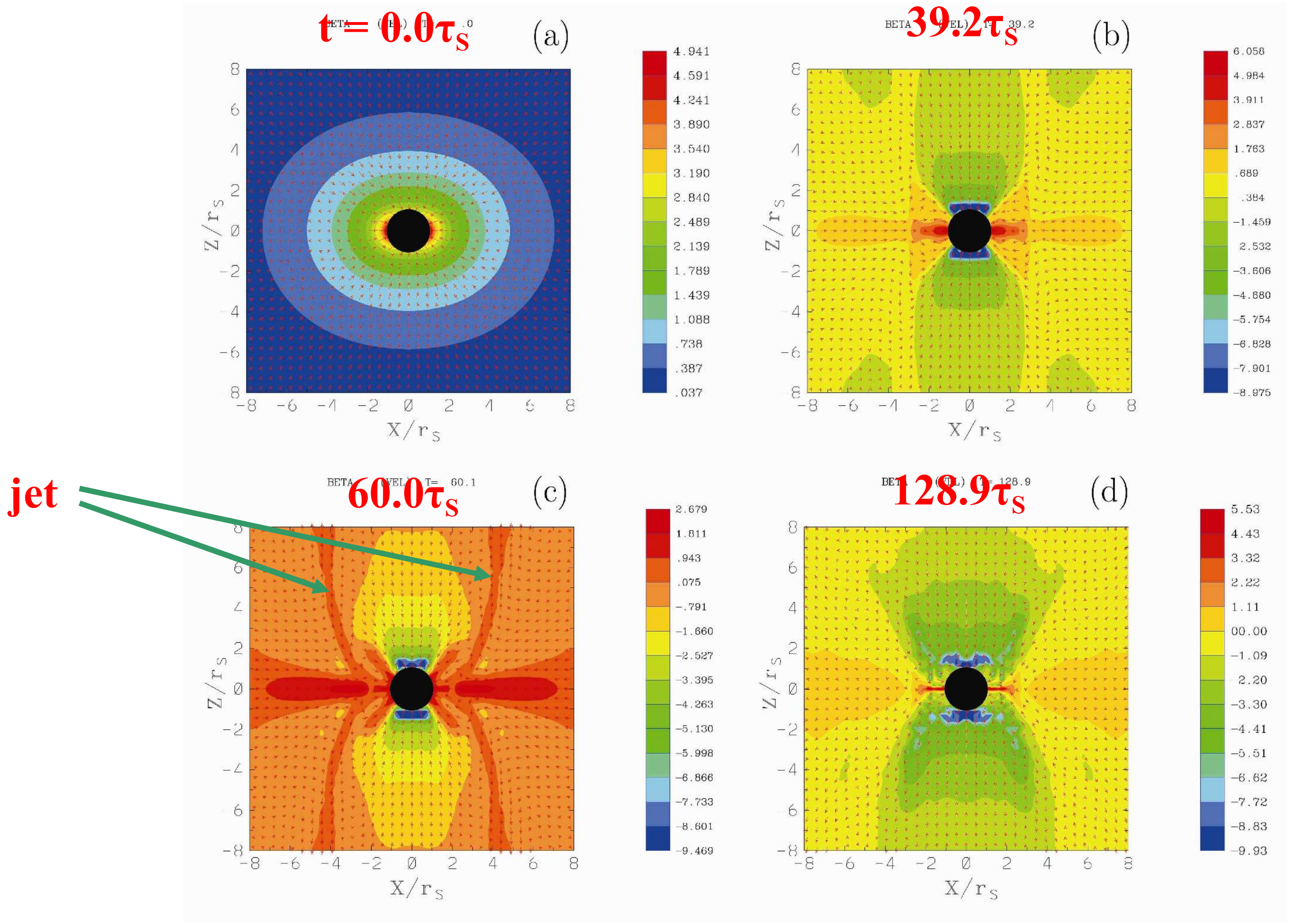
Pressure ($\log_{10}$ p)

falling corona



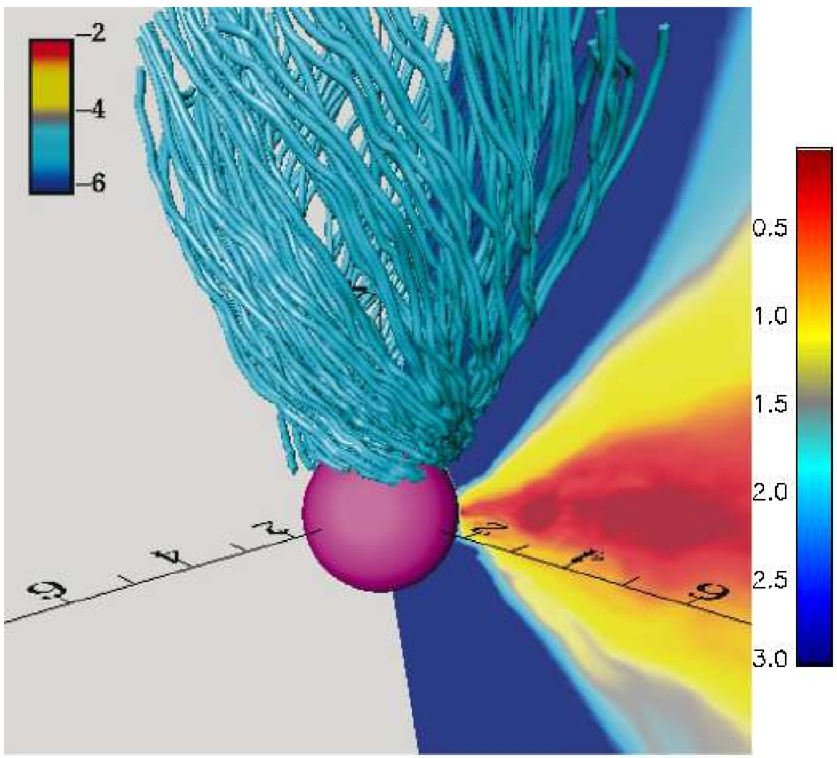
● Blac

Plasma pressure to magnetic pressure



Kerr BH with an initial weak poloidal magnetic fields

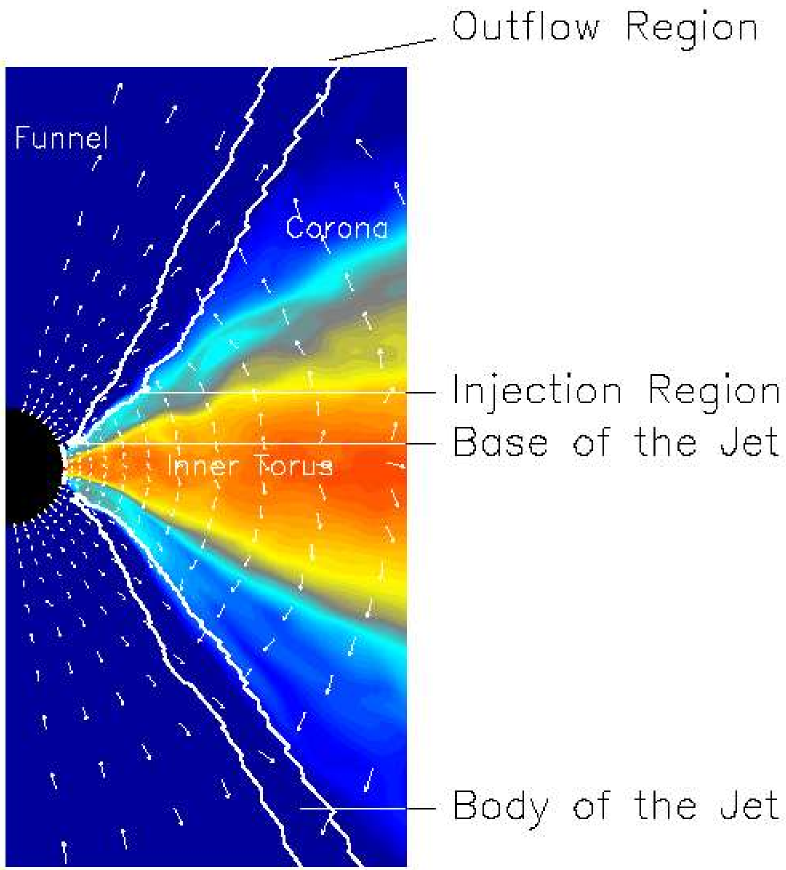
a/M =0.9, t= 7760, 0.1π<θ< 0.2π



log(ρ)

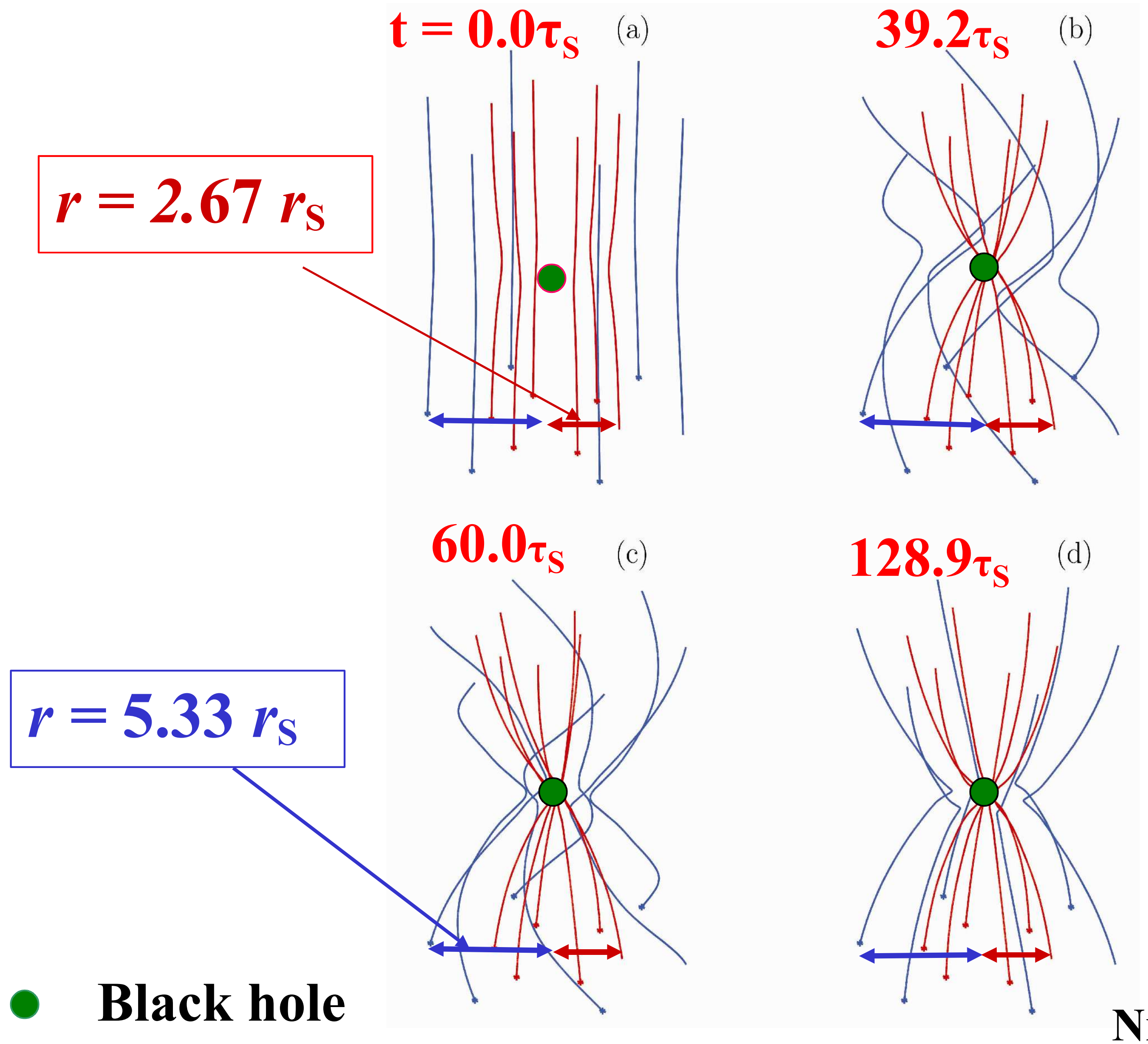
(Hirose et al. 2004)

gas density (log(ρ))



(De Villiers et al. 2005)

Twisted magnetic fields by accretion disk

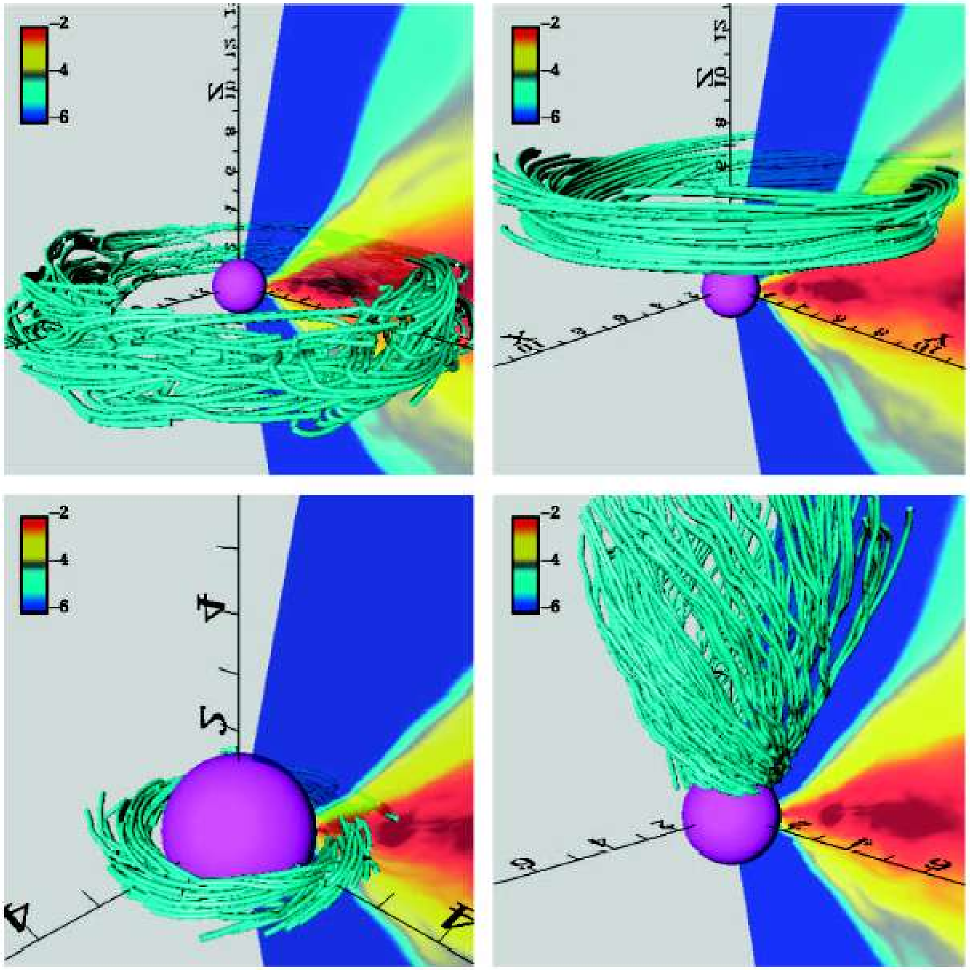


Nishikawa et al (2005)

Sample field lines

the disk body

corona

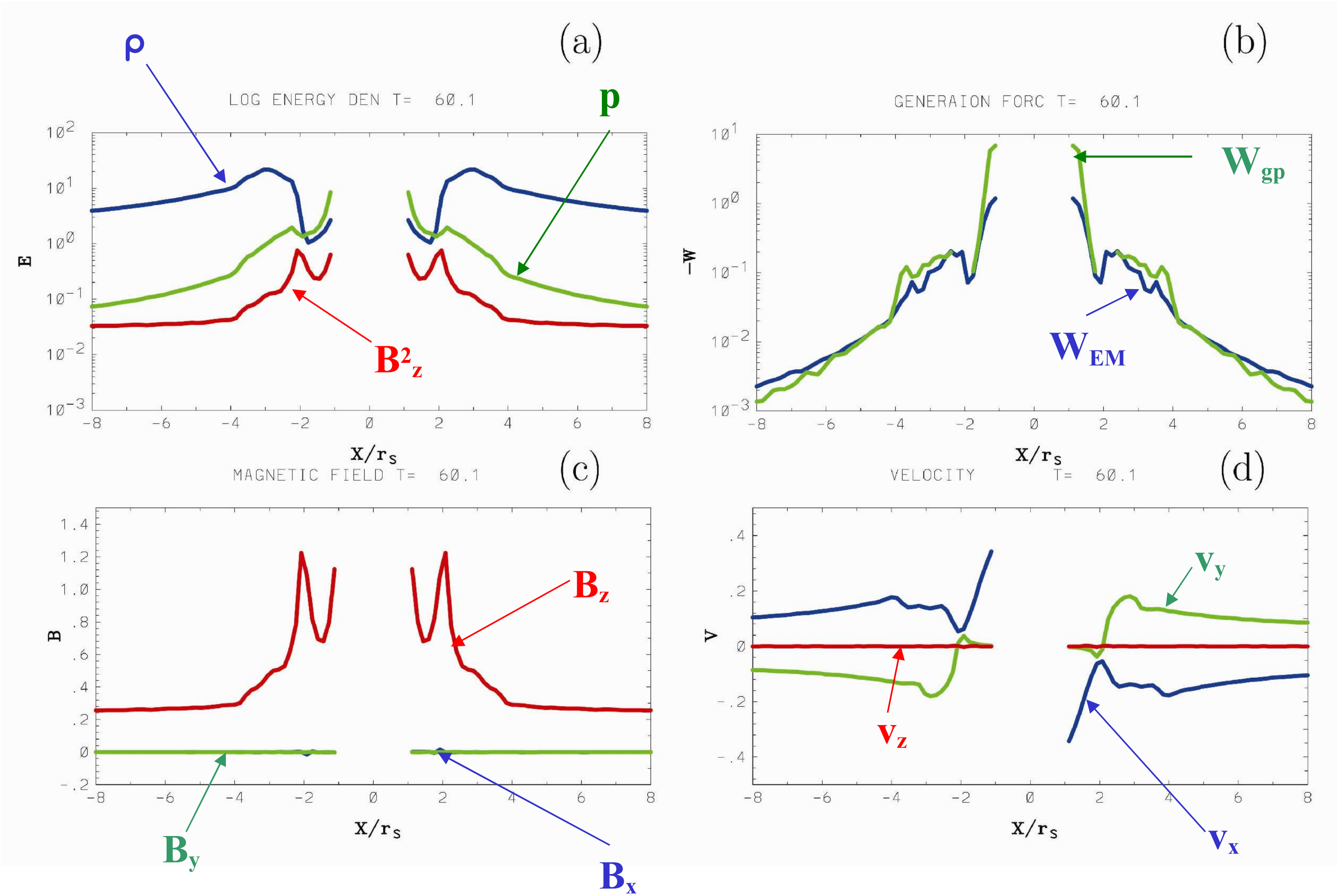


plunging region

axial funnel

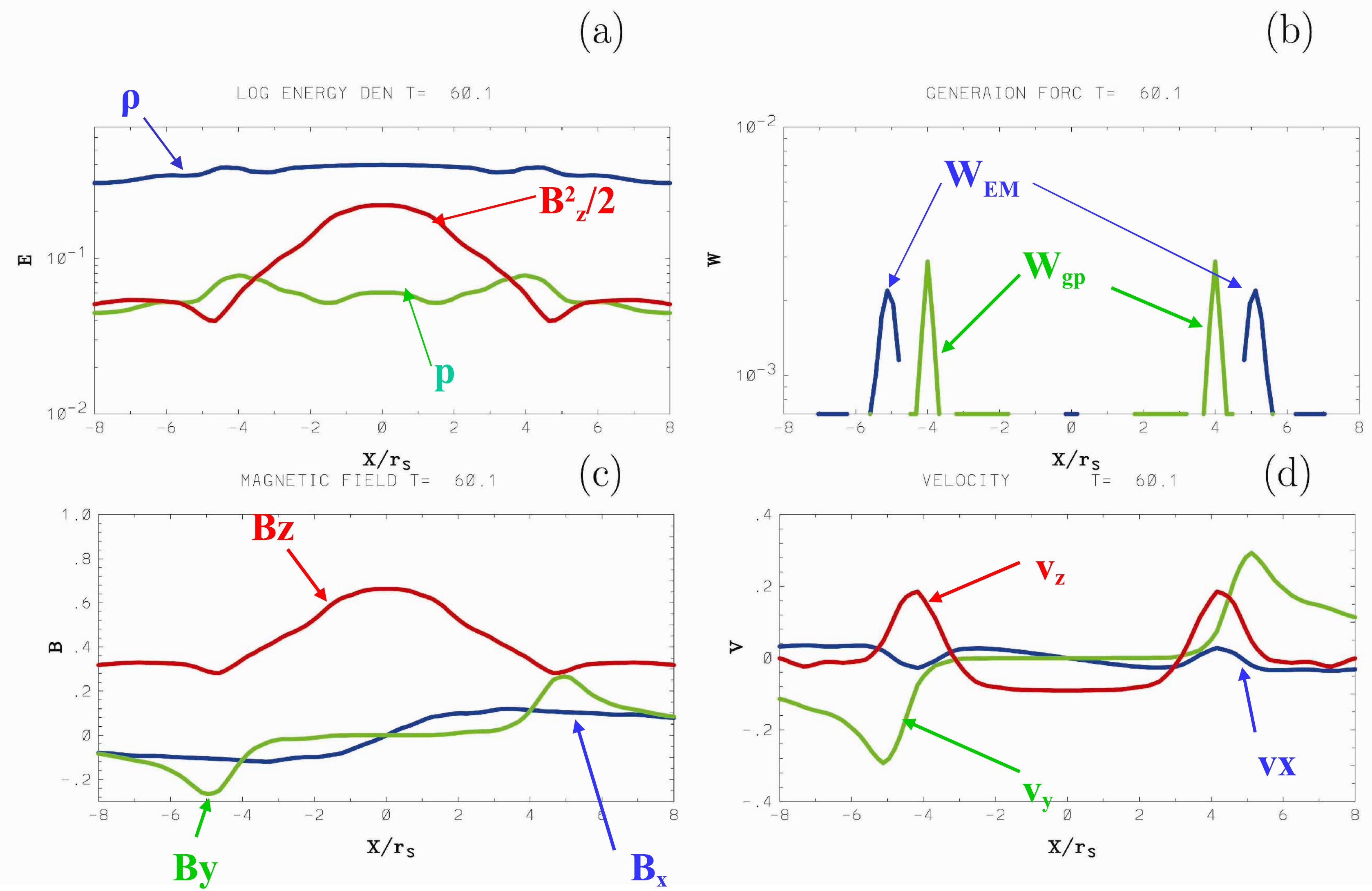
(Hirose at al. 2004)

Snapshots at $z = 0$, $t = 60\tau_S$



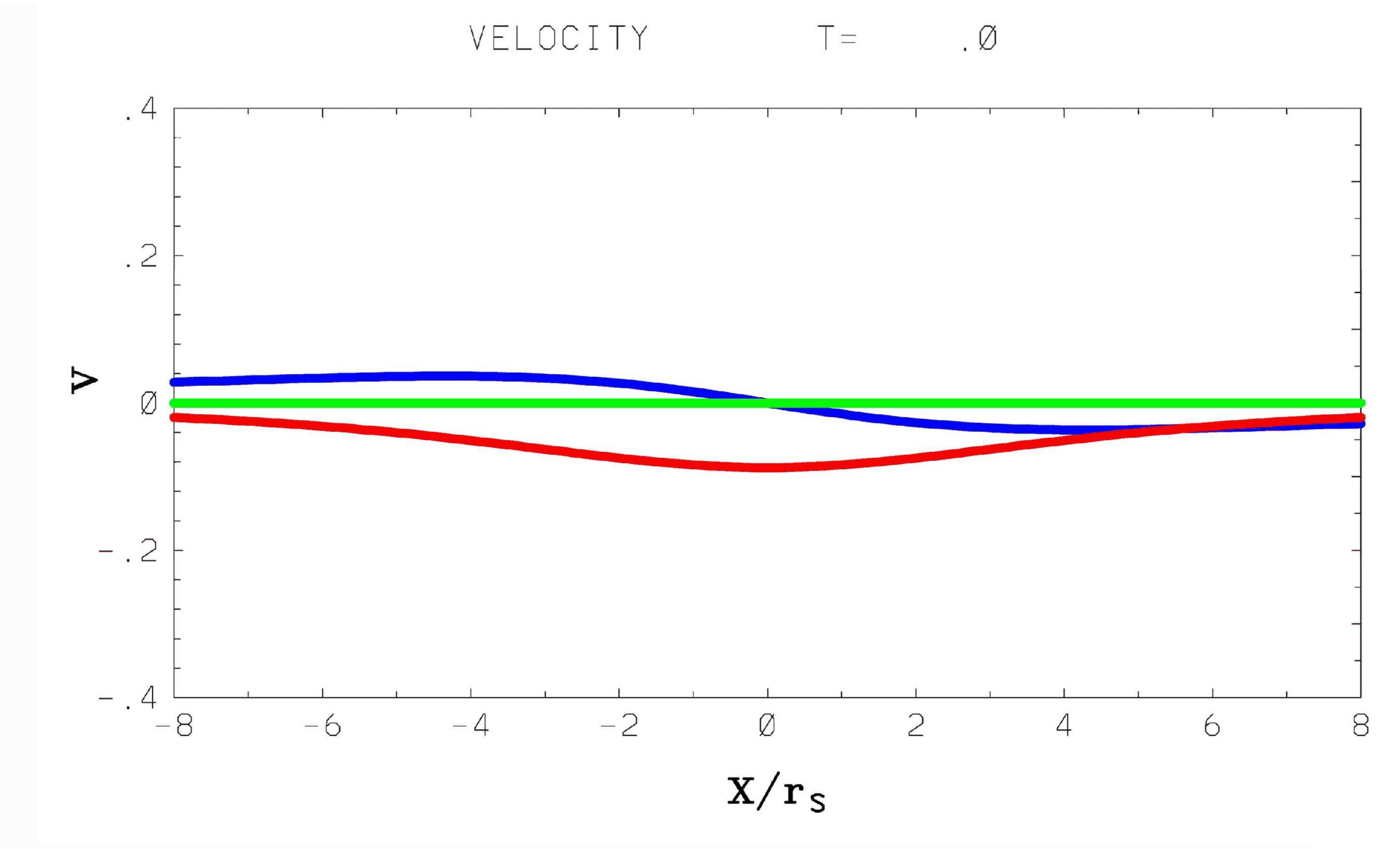
Snapshots at z = 5.6 r_s

$t = 60\tau_s$

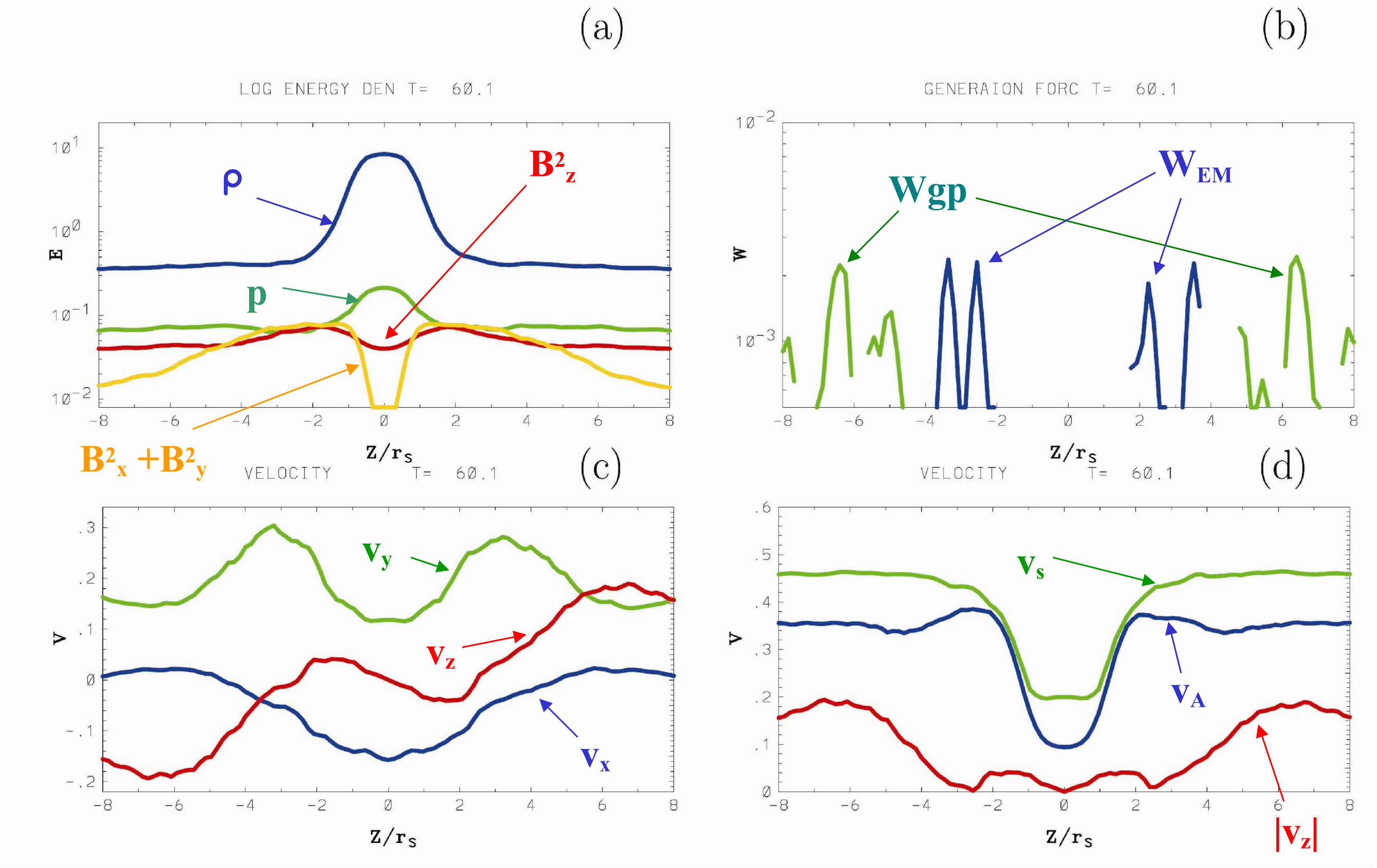


Velocities at Z = 5.6r_S

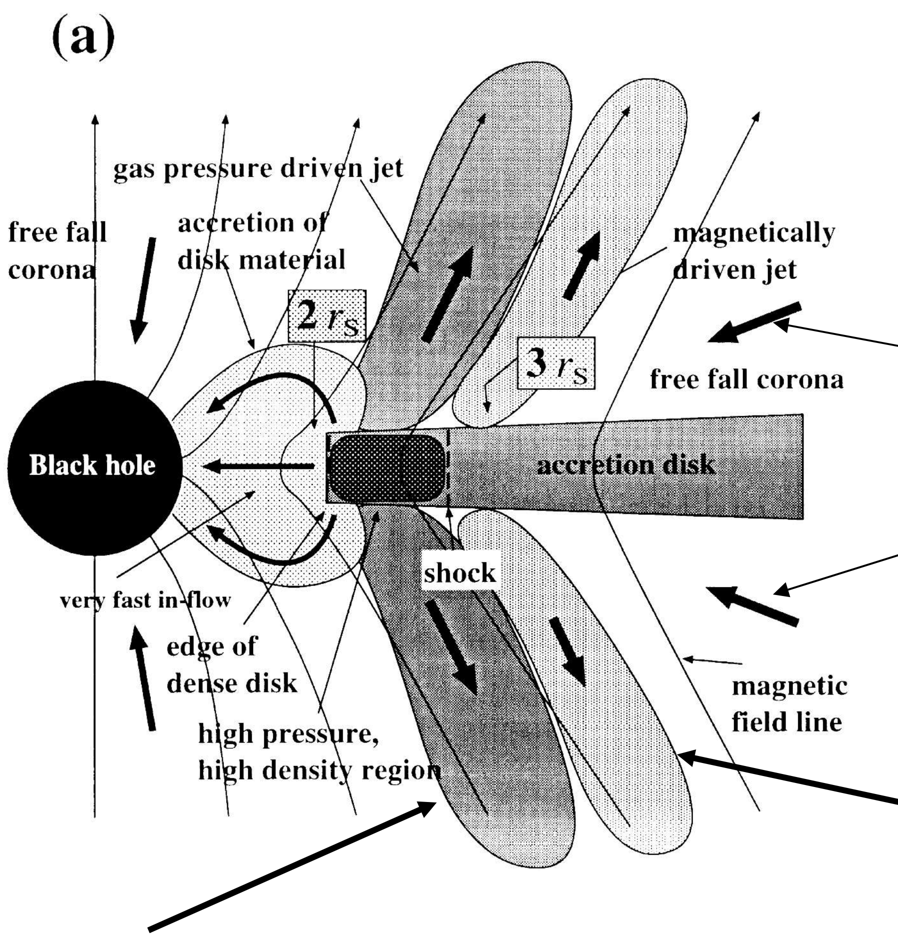
movie



Snapshots at x = 4.48r_s



Schematic picture of two-layer shell structure of relativistic jet



$V_{jet} \approx 0.3\text{-}0.4c$

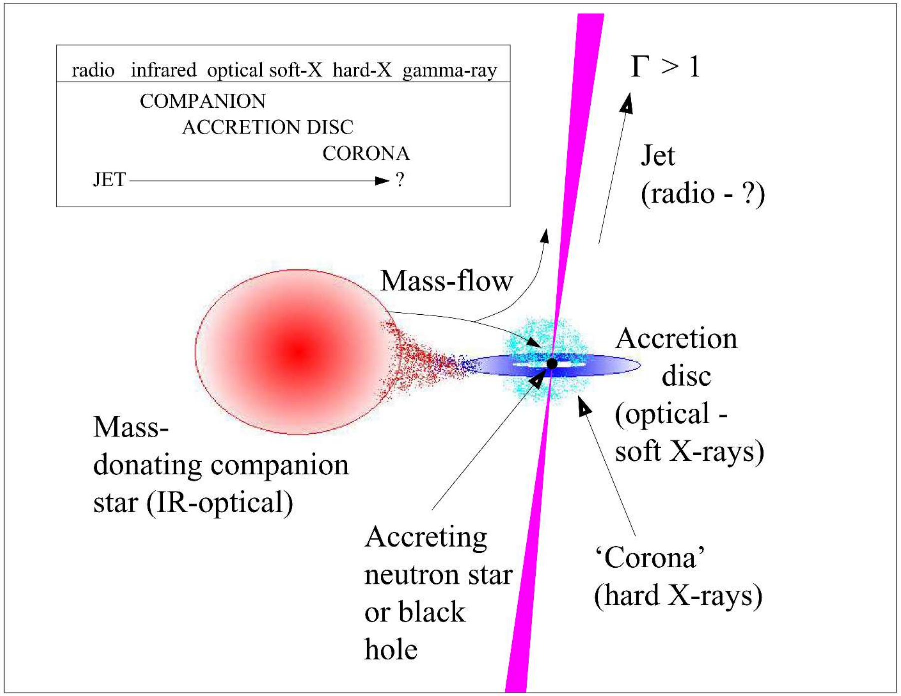
Free-falling corona

magnetically driven jet

gas pressure driven jet

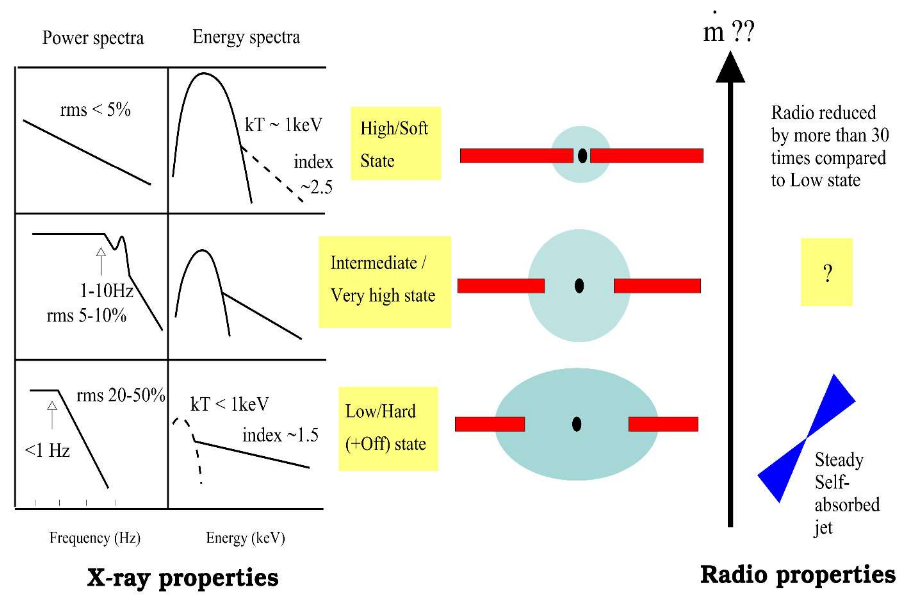
(Koide et al. 1999)

X-ray binary system



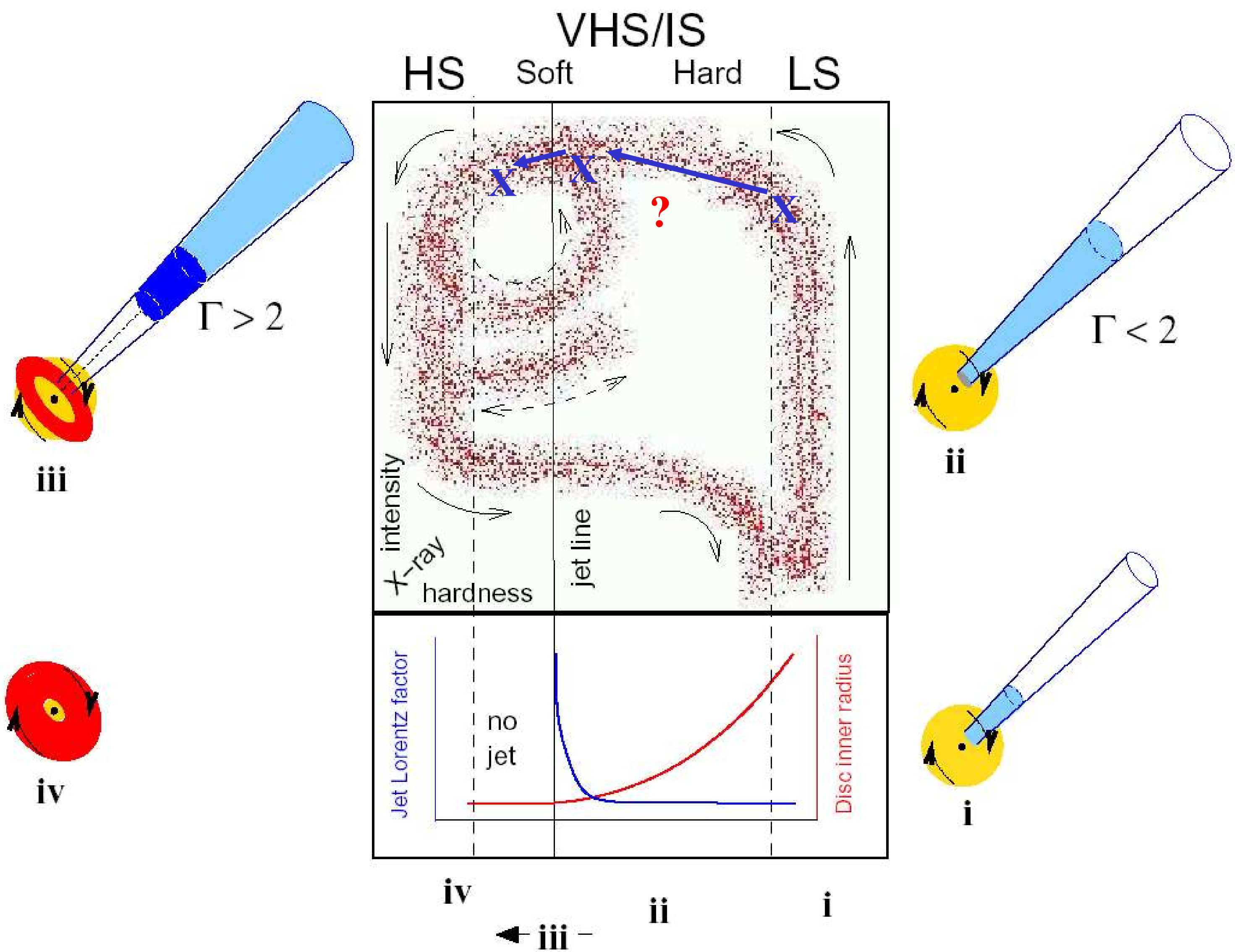
States of BHCs

(Fender 2001)



- ***How does the mass accretion rate determine the state?***
- ***How does the angular momentum, j affect the state?***
- ***How is instability involved with the state transition?***

A schematic model for the jet-disc coupling in black hole binaries

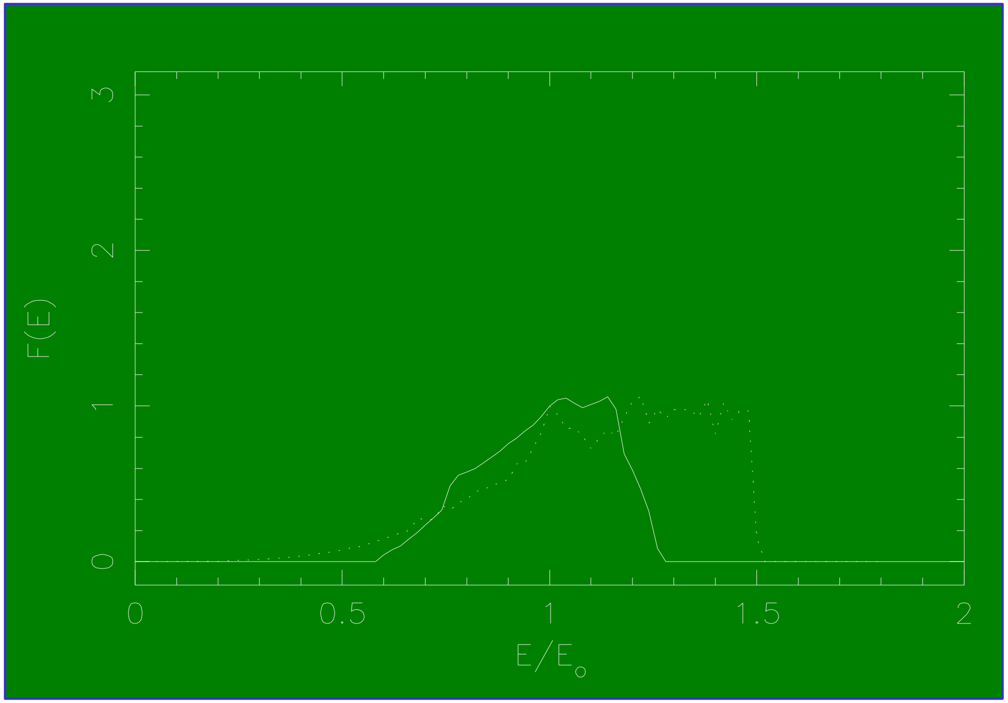


(Fender, Belloni, & Gallo 2004)

Summary

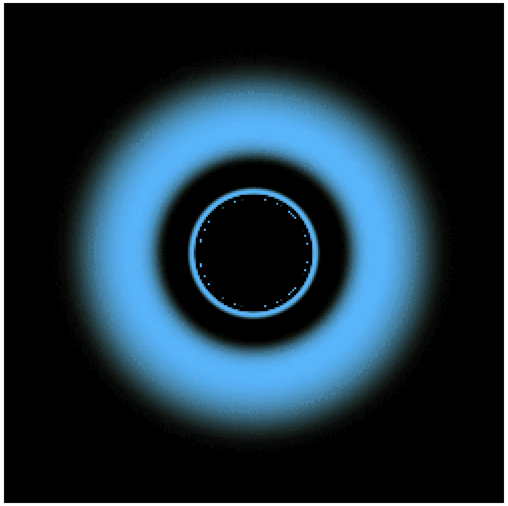
- Comparing the axisymmetric (2-D) simulations 3-D simulation show slower growth of jet formation
- The additional freedom in the azimuthal direction without the mirror symmetry at the equatorial plane slows down the pile-up due to shocks near the black hole
- The longer simulation shows the fading jet and switching to the wind
- In order to see effects of instabilities we need to seed initial perturbations with accretion disks (in progress)

Emission Lines from Tori



- Inner most region is obscured – weakening the red and blue wings.
- A broad emission line centered at 6.4keV results.
- This may explain why not many sources with asymmetric lines like MCG-6-30-15 are observed.

Comparison of emission line between accretion torus (solid) and disk (dotted) inclined at 85°.



Partially transparent torus around a Kerr Black Hole.

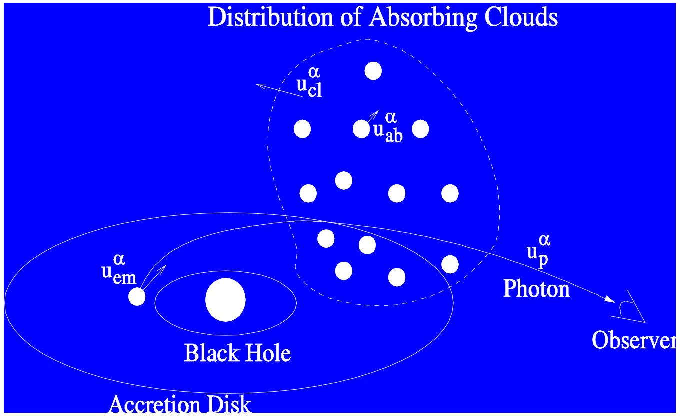
(Fuerst & Wu 2004, A&A, 424, 733)

Relativistic radiative transfer

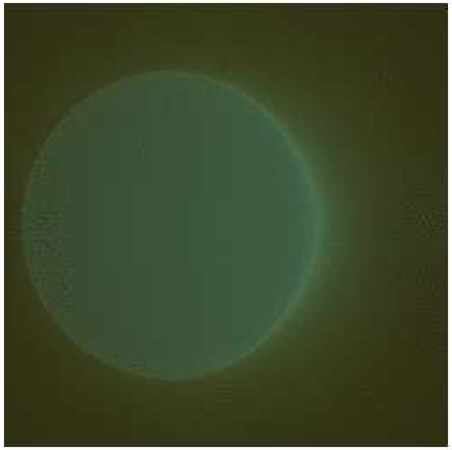
General Relativistic Effects

- Light Bending
  Gravitational Lensing
- Multiple Images
- Gravitational Redshift
- Frame Dragging

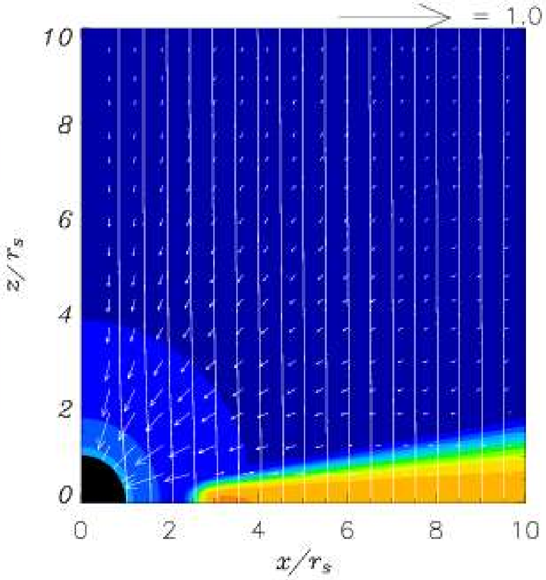
Emission, absorption & scattering



2-D, a = 0.95, B = 0.01 $(\rho c^2)^{-2}$



θ=85 °



Future Plans for jet formation study

- **Investigation of jet generation from Kerr black holes using full 3-D GRMHD simulations with better resolutions and for a long time with a new code**
- **In order to investigate different states of black holes, we will examine how the accretion disk dynamics and associated jet formation depend on initial conditions including magnetic field geometries and accreting stream from mass donor stars**
- **Improve 3-D displays in order to understand physics involved in simulations using IDL, Open DX and AVS Express**
- **Implement a better boundary condition at the horizon**
- **Investigate dynamics of the inner accretion disk near black holes in comparison with observations by Chandra, BATSE, XMM, INTEGRAL, ASTRO E2, GLAST, and Constellation-X**
- **Investigate the dynamics of collapsars as an energy source for Gamma-ray bursts**

Future work

- Micro physics
  - the essential micro physics   physical EOS, treatment of neutrinos
  - It is necessary to include these physics to make reliable quantitative predictions
- Magnetic field configuration
  - The assumption of uniform magnetic field (Wald solution) may be unrealistic

    Dipole-like magnetic field, radial magnetic field etc.

Basic equations (Baumgarte & Shapiro, 2003, ApJ, 585, 921, Duez et al. 2005)

ADM form $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$

Einstein eq

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu} \quad \Rightarrow$$

$$\left.\begin{aligned} R - K_{ij}K^{ij} + K^2 &= 16\pi T_{\mu\nu} n^\mu n^\nu \\ D_i K^i_j - D_j K &= -8\pi T_{\mu\nu} n^\mu \gamma^\nu_j \end{aligned}\right\} \text{Constraints}$$

$$\dot{K}_{ij} = \alpha\left(R_{ij} + KK_{ij} - 2K_{il}K^l_j\right) - D_i D_j \alpha + K_{il} D_j \beta^l + K_{jl} D_i \beta^l + \beta^l D_l K_{ij} - 8\pi\alpha T_{\mu\nu}\left[\gamma^\mu_i \gamma^\nu_j - \frac{1}{2}(\gamma^{\mu\nu} - n^\mu n^\nu)\gamma_{ij}\right]$$

$$\dot{\gamma}_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

Stress-energy tensor

$$\boldsymbol{T}^{\mu\nu} = p g^{\mu\nu} + (e_{in} + p)\boldsymbol{U}^\mu \boldsymbol{U}^\nu + \boldsymbol{F}_\sigma{}^\mu \boldsymbol{F}^{\nu\sigma} - g^{\mu\nu} \boldsymbol{F}^{\lambda\kappa}\boldsymbol{F}_{\lambda\kappa}/4$$

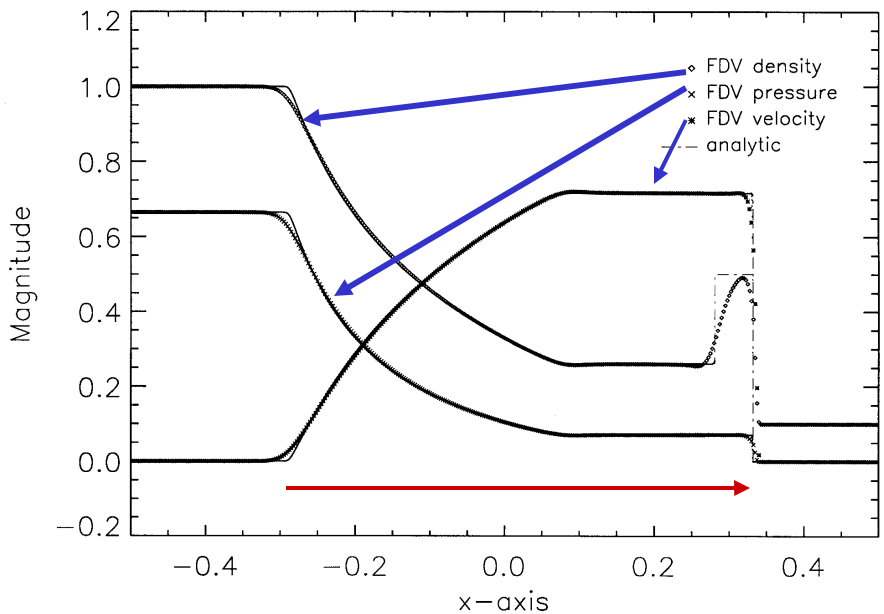
Magnetohydrodynamic equations

Conservation of baryon number	$$\partial_t \rho_* + \partial_j(\rho_* v^j) = 0 \ , \qquad (41)$$
energy equation	$$\partial_t \tilde{\tau} + \partial_i(\alpha^2 \sqrt{\gamma} T^{0i} - \rho_* v^i) = s \ , \qquad (42)$$
energy-momentum conservation equation	$$\partial_t \tilde{S}_i + \partial_j(\alpha \sqrt{\gamma} T^j{}_i) = \frac{1}{2} \alpha \sqrt{\gamma} T^{\alpha\beta} g_{\alpha\beta,i} \ , \qquad (43)$$
induction equation **(fozen-in condition)**	$$\partial_t \tilde{B}^i + \partial_j(v^j \tilde{B}^i - v^i \tilde{B}^j) = 0 \ . \qquad (44)$$

Flowfield Dependent Variation Method (FDV) in Finite Element Form for Shock Capturing in Relativistic Environments

Richardson & Chung ApJS 139, 539

Relativistic Shock Tube

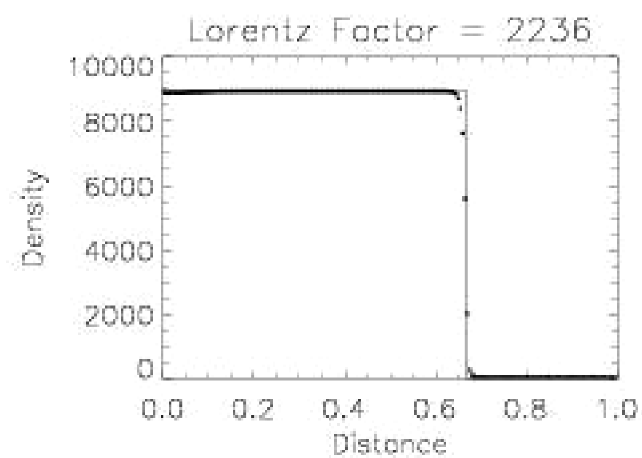


(2-D GRHydro simulations)

Finite Element Method:

- **Allow unstructured grids and complex geometries**
- **Integral form improves application of flux boundary conditions over FDM**

Relativistic Reflective Shock



Flowfield Dependent Variation Method:

- **Inherent diffusion terms reduce artificial viscosity effects**
- **Physical parameters are used to adjust the solution method**

will be extended to GRMHD coupled with Einstein equations

Fundamental equations

- $\nabla_{\gamma}(\rho U^{\gamma}) = 0$ (**Conservation of mass**)
- $\nabla_{\mu} T^{\mu\nu} = 0$ (**Conservation of momentum**)
- $\partial_{\mu} F_{\alpha\beta} + \partial_{\alpha} F_{\beta\mu} + \partial_{\beta} F_{\mu\alpha} = 0$

  (**Conservation of energy for single component conductive fluid**)
- $\nabla_{\alpha} F^{\alpha\beta} = -J^{\beta}$ (**Maxwell's equations**)

  $F_{\alpha\beta}$ **(electromagnetic field-strength tensor)**

  $F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha}$

  $F_{\alpha\beta} U^{\mu} = 0$ (**Frozen-in condition**)

$\boldsymbol{U}^{\gamma}$: velocity 4-vector

$\boldsymbol{J}^{\gamma}$: current 4-vector

ρ : proper mass density

p : proper pressure

$e_{\rm in} = \rho\, c^2 + p/(\Gamma - 1)$: energy density

Γ : specific-heat ratio (5/3)

∇_{γ} : covariant derivative

$\boldsymbol{T}^{\mu\nu} = pg^{\mu\nu} + (e_{\rm in} + p)\boldsymbol{U}^{\mu}\boldsymbol{U}^{\nu} + \boldsymbol{F}_{\sigma}^{\ \mu}\boldsymbol{F}^{\nu\sigma} - g^{\mu\nu}\boldsymbol{F}^{\lambda\kappa}\boldsymbol{F}_{\lambda\kappa}/4$:

general relativistic energy momentum tensor

$\boldsymbol{A}^{\mu}$: potential 4-vector

3+1 Formalism of General Relativistic MHD Equations

$\partial D/\partial t = -\nabla\cdot(D\boldsymbol{v})$

$\partial \boldsymbol{P}/\partial t = -\nabla\cdot[p\boldsymbol{I}+\gamma^{2}(e+p)\boldsymbol{vv}/c^2-\boldsymbol{BB}-\boldsymbol{EE}/c^2+0.5*(B^2+E^2/c^2)\boldsymbol{I}]$

$\partial\varepsilon/\partial t = -\nabla\cdot[\{\gamma^{2}(e+p)-D^2c^2\}\boldsymbol{v}+\boldsymbol{E}\times\boldsymbol{B}]$

$\partial \boldsymbol{B}/\partial t = -\nabla\times\boldsymbol{E}$

$(1/c^2)\,\partial \boldsymbol{E}/\partial t + \boldsymbol{J} = -\nabla\times\boldsymbol{B}$ **(replacement current)**

$(1/c^2)\,\nabla\cdot\boldsymbol{E} = \rho_{\mathrm{c}}$ $\nabla\cdot\boldsymbol{B} = 0$ **(constraint)**

$\boldsymbol{E} = -\,\boldsymbol{v}\times\boldsymbol{B}$ (Frozen-in condition)

$\gamma \equiv [1-(v/c)^2]^{-1/2}$, $D = \gamma\rho$, $\boldsymbol{P} = \gamma^2(e+p)\boldsymbol{v}/\mathrm{c}^2 + \boldsymbol{E}\times\boldsymbol{B}/\mathrm{c}^2$

$\varepsilon = \gamma^{2}(e+p) - p - Dc^2+0.5*(B^2+E^2/\mathrm{c}^2)$

Metric and Coordinates

Schwarzschild metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

Boyer-Lindquist set (ct, r, θ, φ)

Off-diagonal elements of metric are zero

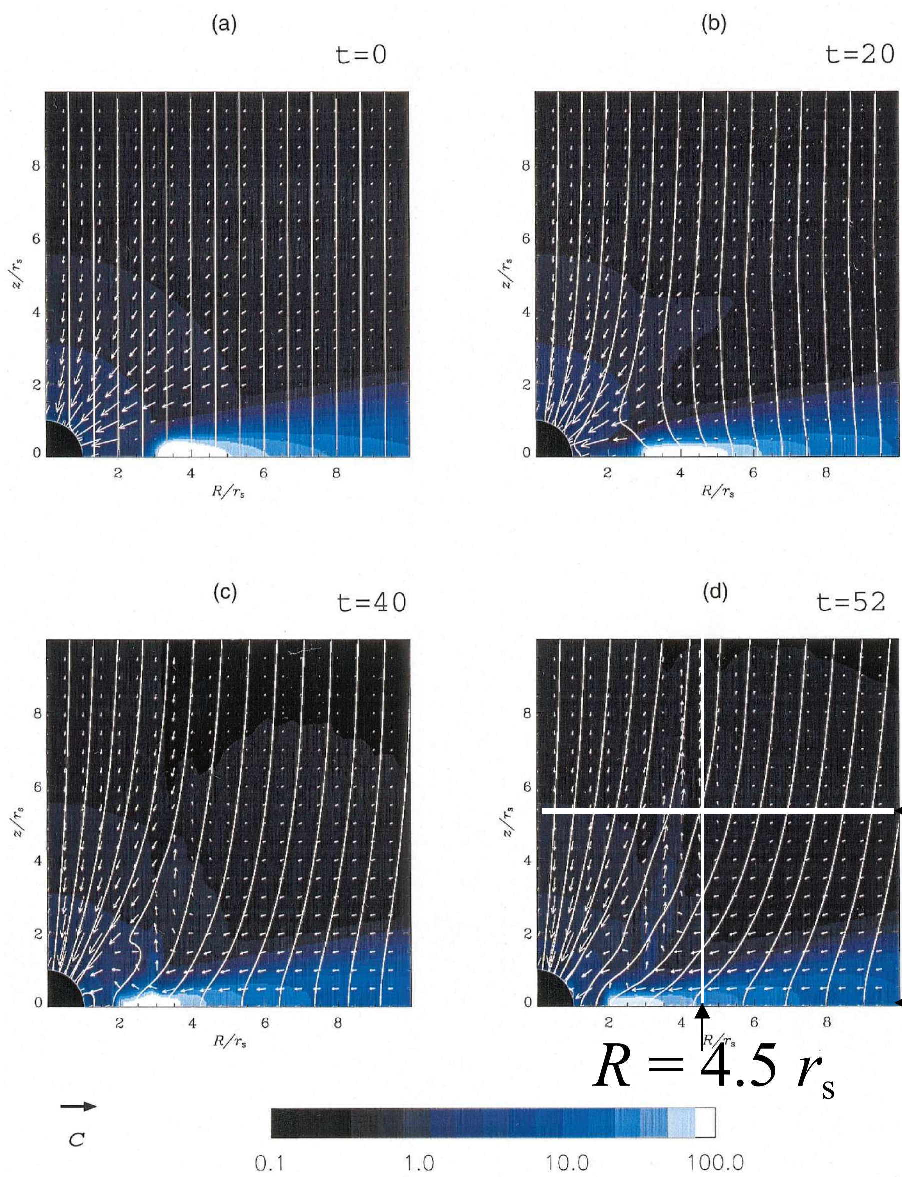
$$g_{\mu\nu} = 0 \quad (\mu \neq \nu)$$

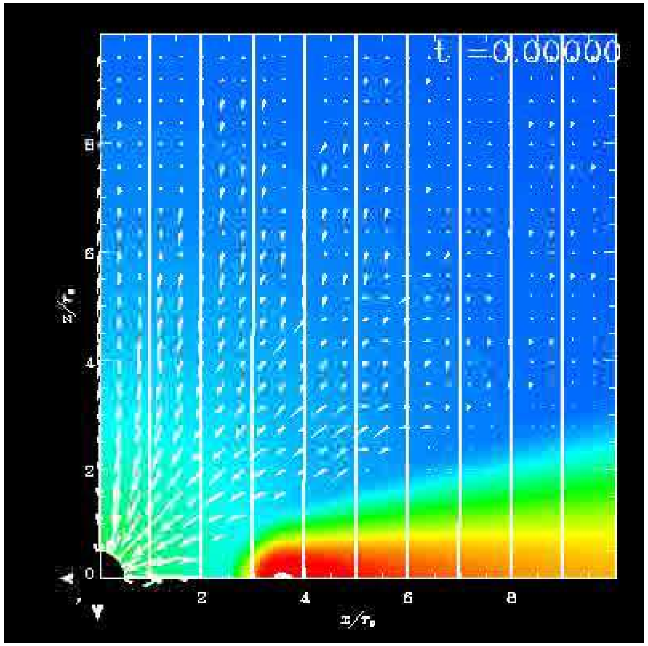
$$g_{00} = -h_0^2, \; g_{11} = h_1^2, \; g_{22} = h_2^2, \; g_{33} = h_3^2$$

$$h_0 = \alpha, \; h_1 = 1/\alpha, \; h_2 = r, \; h_3 = r \cos\theta$$

$$\alpha \equiv (1 - r_S/r)^{1/2} \quad \text{(lapse function)}$$

2-D axisymmetric simulation





Movie

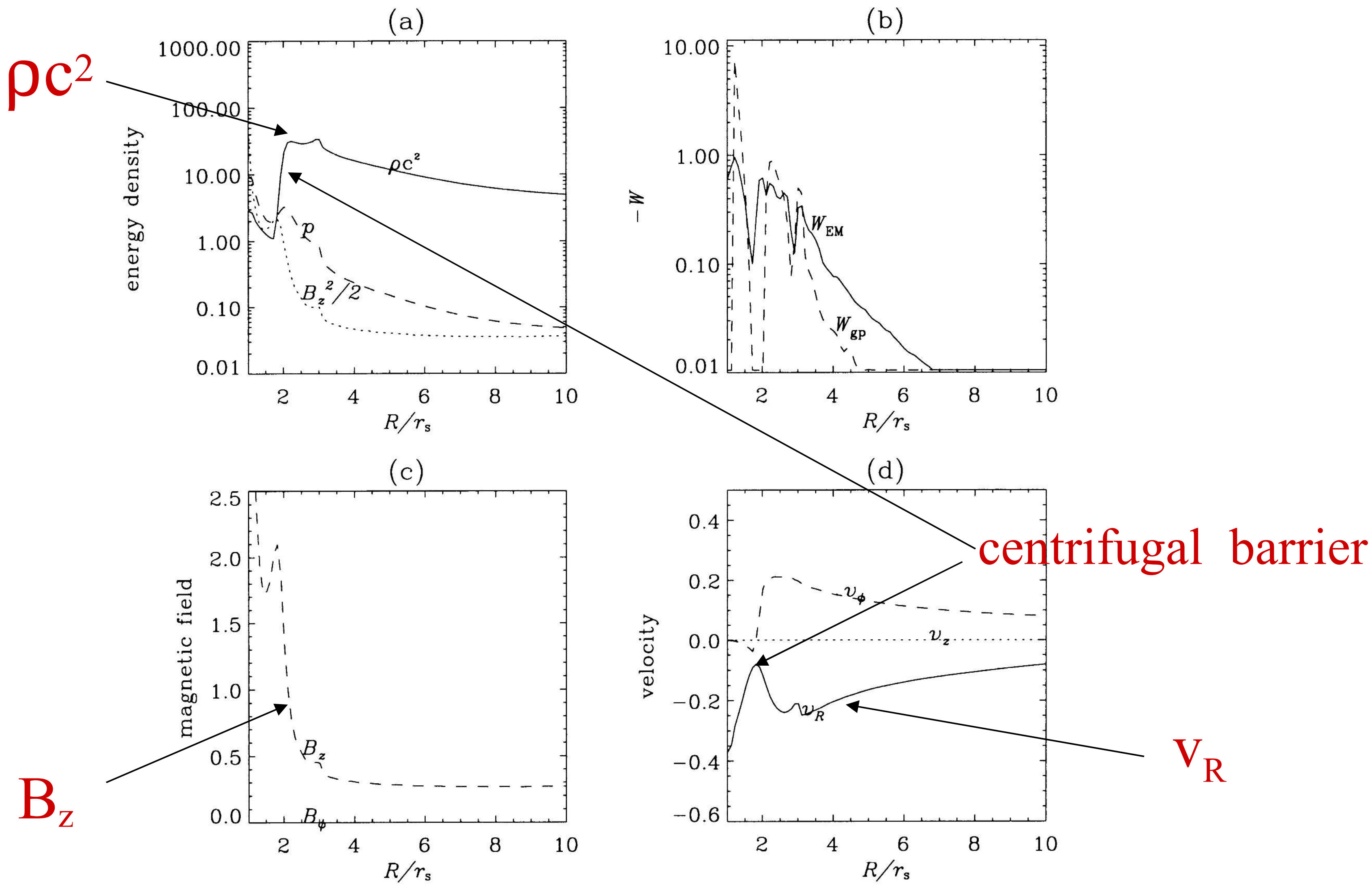
$z = 5.6\ r_s$

$z = 0.0 r_s$

$R = 4.5\ r_s$

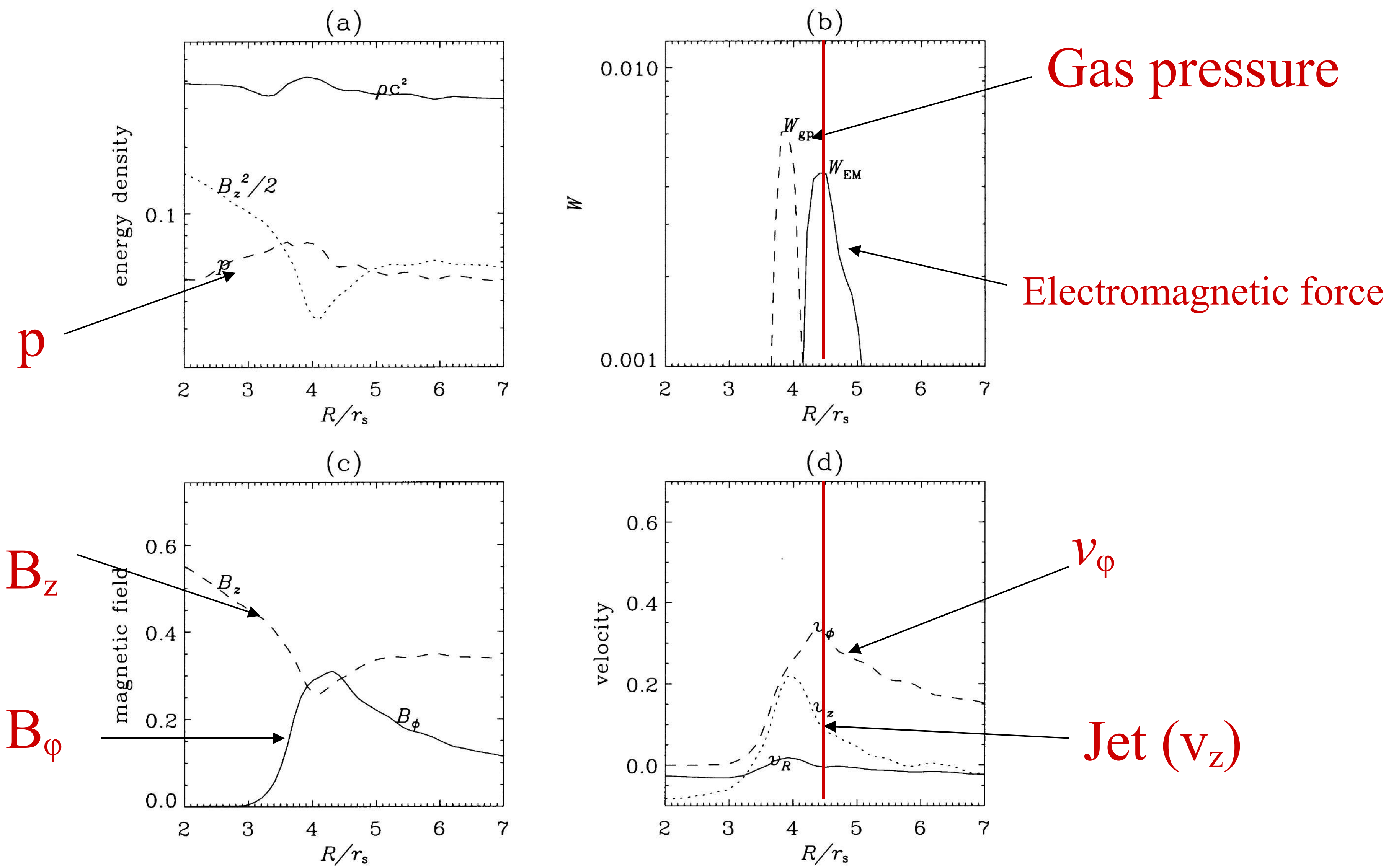
(Koide et al. 1999)

Radial Profiles, equatorial plane ($z = 0$), $t=52\ \tau_S$



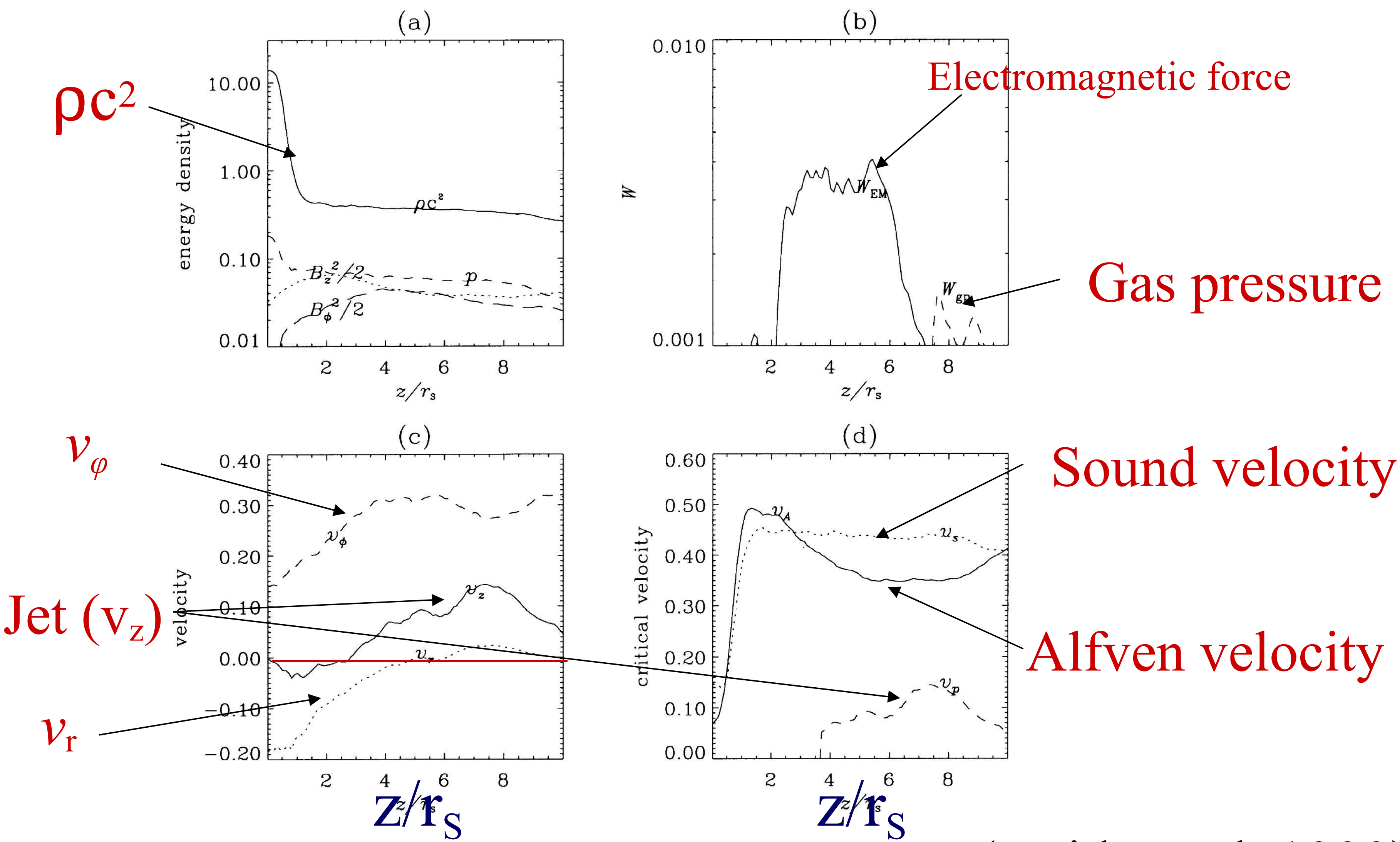
(Koide et al. 1999)

Radial profiles, ($z = 5.6\ r_s$), $t=52\ \tau_S$



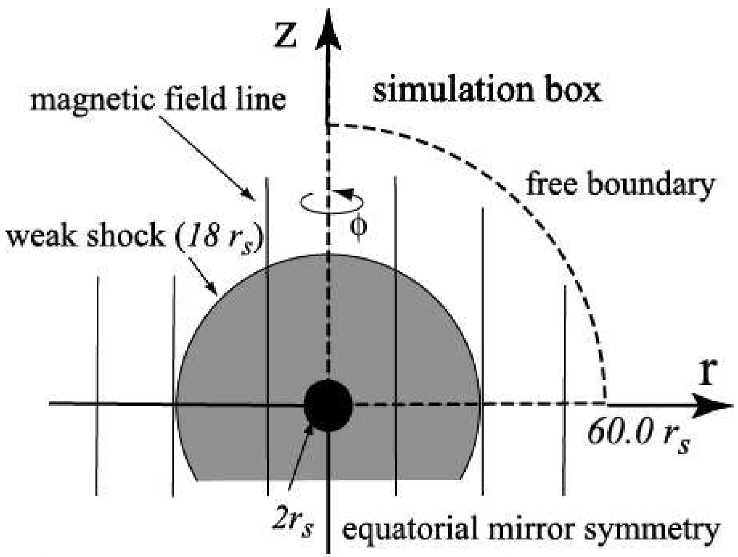
(Koide et al. 1999)

Z-profiles, ($R = 4.5\ r_s$), t=52 τ_S



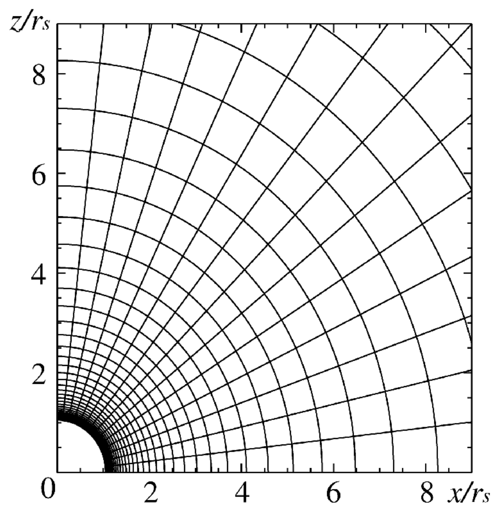
(Koide et al. 1999)

2-D GRMHD simulations by Mizuno et al. 2004 (Kerr BH)



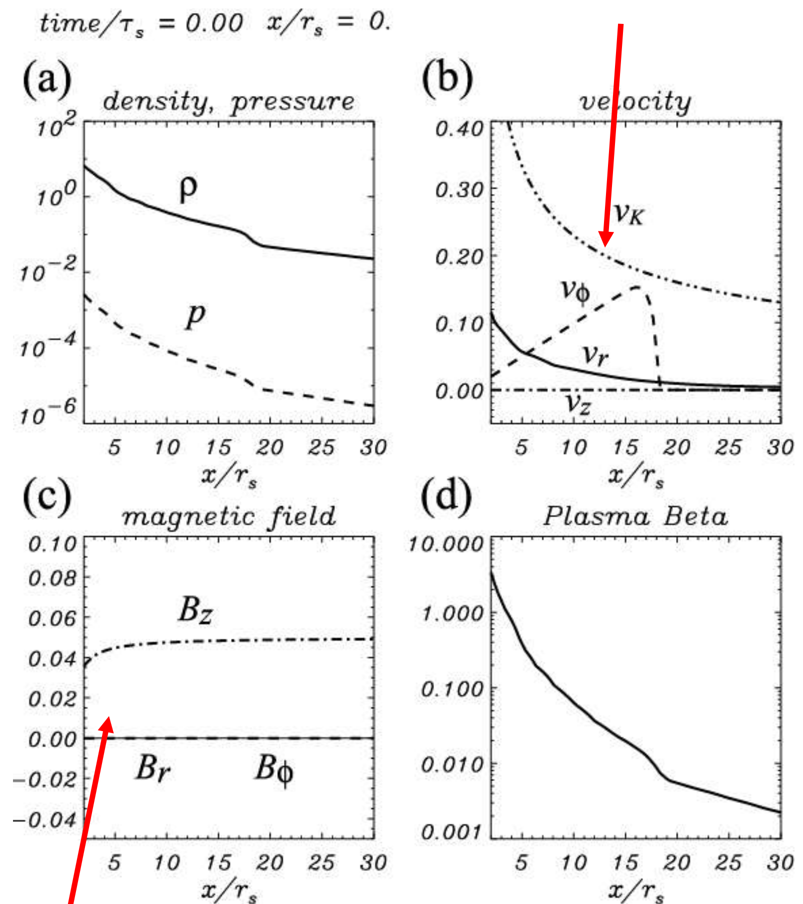
Schematic picture of our simulation

Gray: rotation



Distribution of mesh point

Rigid-like rotation



Uniform magnetic field

The distribution on equatorial plane

Bruenn, 1992

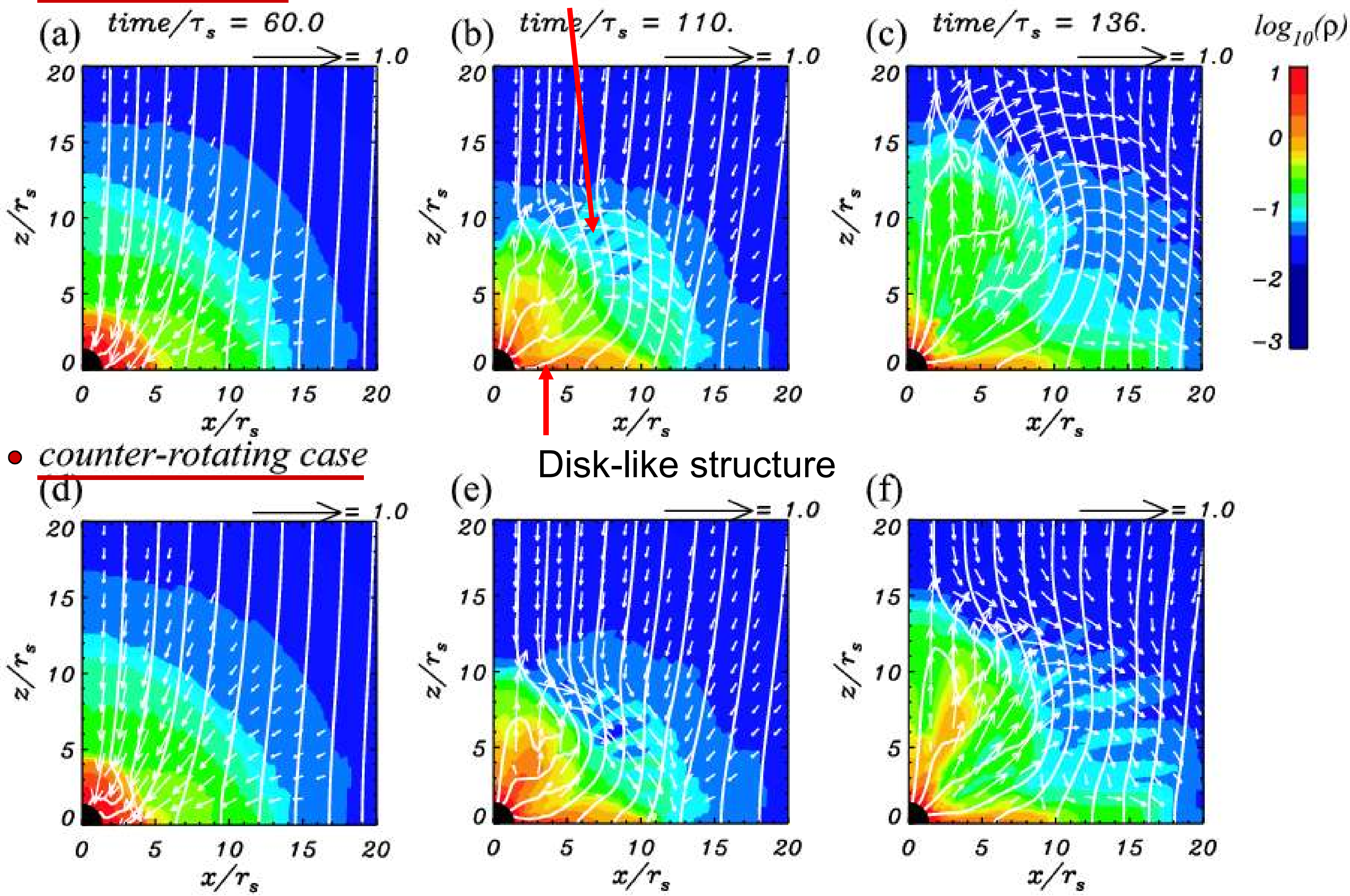
Density in 2-D GRMHS simulations (Kerr BH)

color density
line magnetic field lines

$\tau_s = r_s/c$

Jet-like ejection
near the central BH

- co-rotating case
- counter-rotating case



Disk-like structure

Summary Kerr BH case

- The formation mechanism of the jet in the rotating BH case is the same as that of the non-rotating BH case (mainly by the magnetic field)
- The jet velocity in the co-rotating case is comparable to that of the non-rotating case, v_{jet} 0.3c
- In the co-rotating case, the kinetic energy flux is comparable to the Poynting flux
- As the rotation parameter of BH increases, the poloidal velocity of the jet and magnetic twist increase gradually and toroidal velocity of the jet decreases. Because the magnetic field is twisted strongly by the frame dragging effect, it can store much magnetic energy and converts to kinetic energy of the jet directly

Motivations

- **Jet formation from black holes**
- **Accretion disk dynamics including azimuthal instabilities such as magnetorotational instability (MRI) and *accretion-ejection instabilities (AEI)* with various initial and magnetic field geometries**
- **Variabilities of relativistic jets due to these instabilities in the accretion disk and their effects on jet propagations**
- **Modeling high/soft and low/hard states with AGNs related to mass accretion rates and angular momentum of black holes.**
- **Examination of Blandford-Znajek model with a Kerr black hole as a possible energy source for Gamma-ray Bursts?**

Theoretical models of jet formation

Lovelace (1976), Blandford (1976)

Blandford & Znajek (1997): Kerr black hole

Blandford and Payne (1982):

Magneto-centrifugal force-driven jet

Begeleman, Blandford, & Rees (1984):

"Theory of extragalactic radio sources"

Uchida & Shibata (1985): Magnetically driven

Koide, Shibata & Kudoh (1998): (2-D GRMHD)

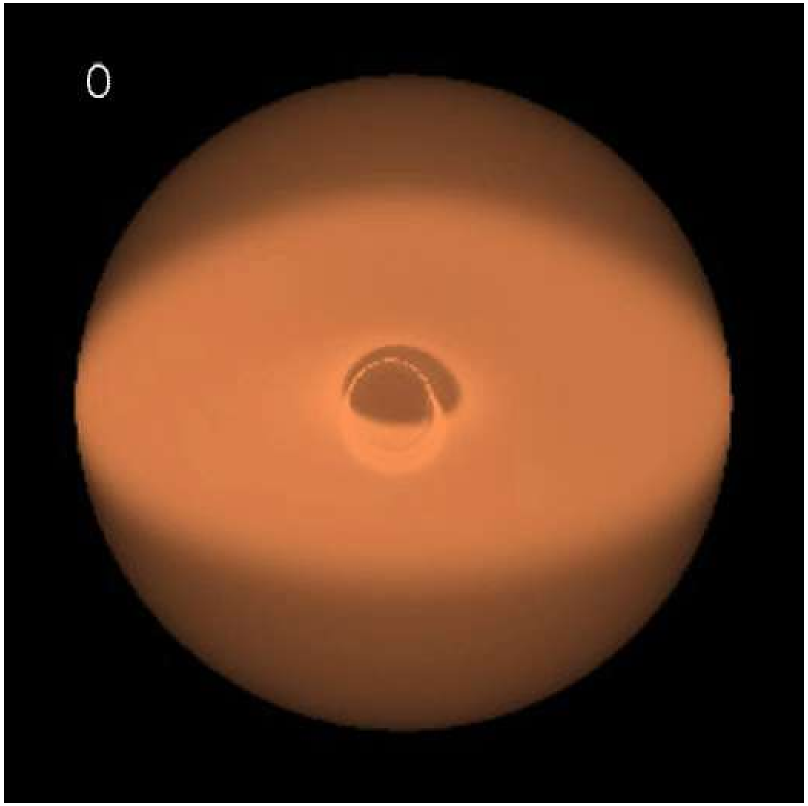
"*Gas pressure*" & Magnetically driven

Relativistic radiative transfer

2-D, a = 0.95, B = 0.03 $(\rho c^2)^{-2}$

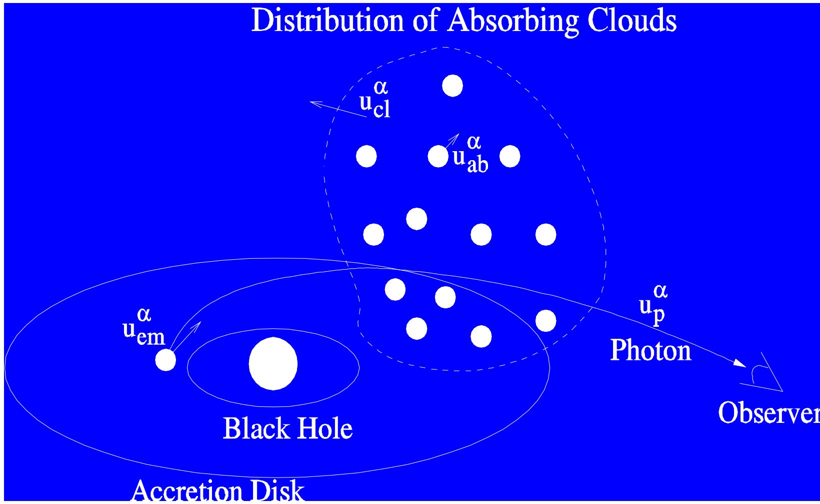
60°

$t/\tau_S =$



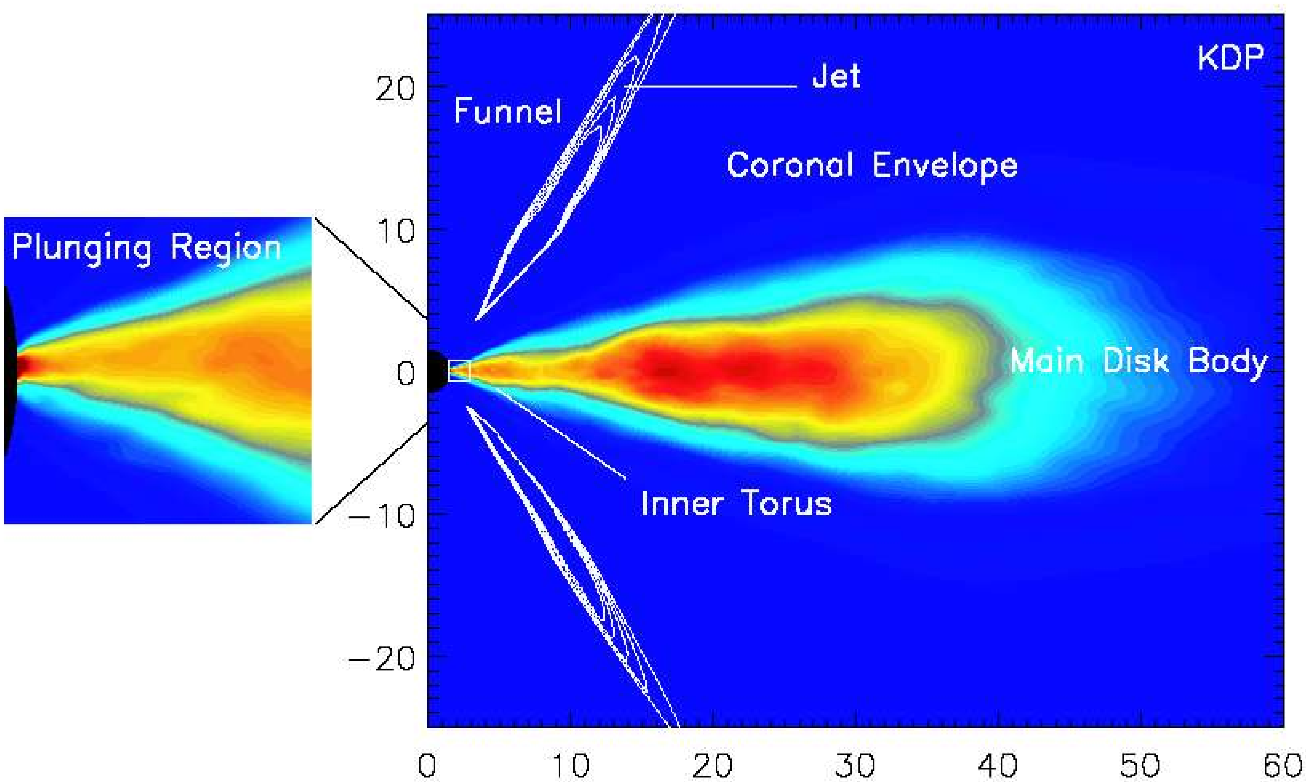
General Relativistic Effects

- Light Bending \ Gravitational Lensing
- Multiple Images
- Gravitational Redshift
- Frame Dragging



Emission, absorption & scattering

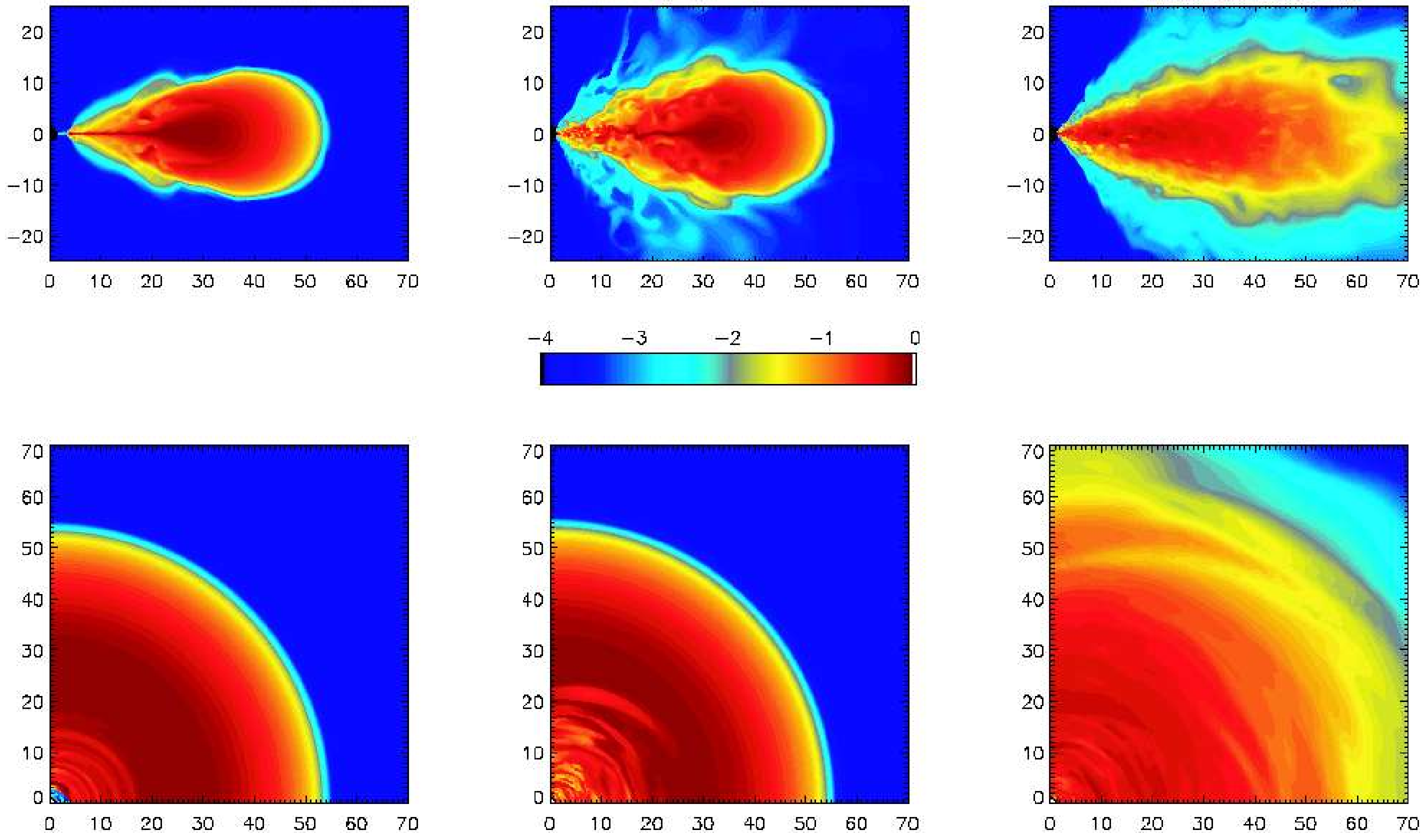
Jet by MRI



(De Villiers, Hawley, & Krolik, 2003c)

MRI simulations

a/M = 0.9



(De Villiers, Hawley, & Krolik, 2003c)