General Relativistic Magnetohydrodynamic Simulations of Collapsars

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Y.Mizuno et al. 2005, ApJ, 606, 395
Y.Mizuno et al. 2005, ApJ, 615, 389

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General Properties of GRBs

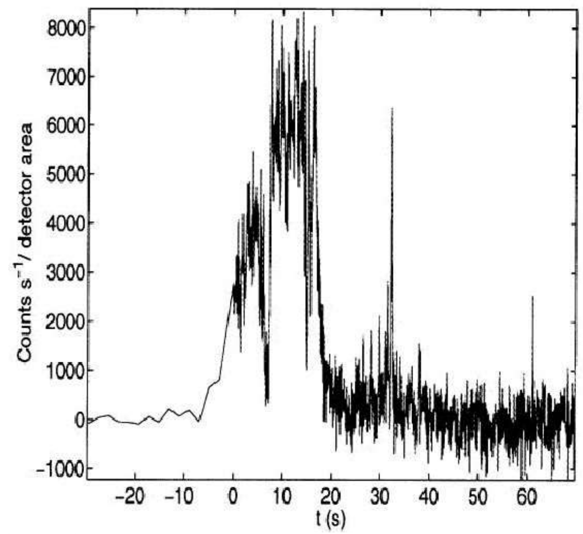
- Gamma-Ray Bursts (GRBs) are known the most energetic explosions
  - Duration (few ms - 1000sec)
    - 2 populations (long-soft, short-hard)
  - Cosmological distance (z 1
  - Isotropic energy= 10^{52}-10^{54} erg but presumed to be highly beamed
- GRBs are relativistic jet 100 ejected from compact central engines
- Conversion to radiation by shock scenario
  - Internal shocks (collision of shells) →GRB (prompt emission)
  - External shocks (collision with ISM) →afterglow emission
- Central engine of GRBs is unknown (The most fundamental problem)

Observational Properties of GRBs

- Gamma-Ray Bursts (GRBs) are known the most energetic explosions
- Duration (few ms - 100sec)
  - Various shape light curves
  - Rapid time variability ms
  - 2 populations (<u>long-soft</u>, short-hard)
- Frequency: few per day
- Cosmological distance (z 1

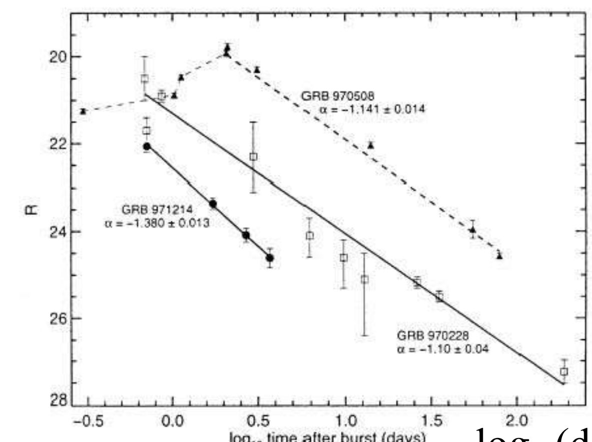
Isotropic energy=10^{52}-10^{54} erg but presumed to be highly beamed

- Afterglows seen after GRB events (long burst only)
  - Power law decay from -ray to radio
  - Continue over 100 days



T(s)

light curve of GRB970228



$\log_{10}$(day)

Afterglow light curve

Fireball Model

Most favored explanation model of GRBs

Shemi & Piran (1990)

In Fireball Scenario

- Compact central engine

→ relativistic outflow ˜ı

→ From compactness problem

Avoid being optically thick

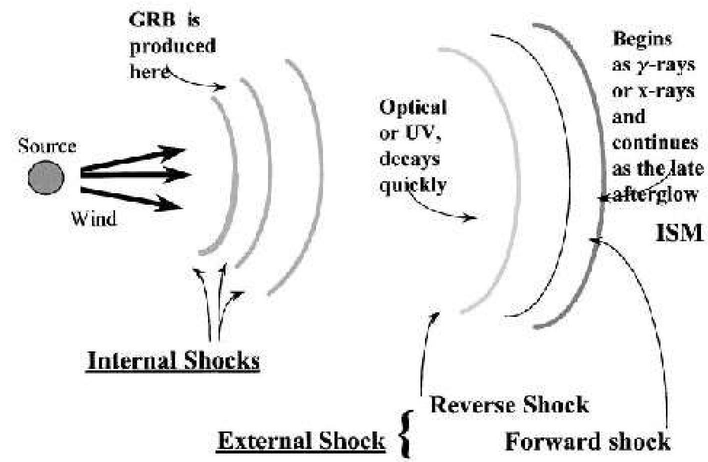
- Conversion to radiation by shock scenario
  - Internal shocks (collision of shells)

  → GRB (prompt emission)
  - External shocks (collision with ISM)

  → afterglow emission

- Central engine of GRBs is unknown

(The most fundamental problem)



Schematic figure of Fireball model

Compactness Problem

- Rapid temporal variability $\delta t \quad 10ms$

$\rightarrow$ source is compact ($R_i < c\delta t \quad 3000km$)

- Spectrum$\rightarrow$ contains a lot of high energy γ-ray photons
- Interaction with low-energy photons $\rightarrow$ e^+e^- pairs
- Average optical depth

$$\tau_{\gamma\gamma} = f_p \sigma_T F D^2 / R_i^2 m_e c^2$$

$$\tau_{\gamma\gamma} = 10^{13} f_p \left(\frac{F}{10^{-7}\, erg/cm^2}\right)\left(\frac{D}{3000\, Mpc}\right)^2 \left(\frac{\delta T}{10 mS}\right)^{-2}$$

- Optical depth is large
- However observed non-thermal spectrum $\rightarrow$ optically thin !

Compactness Problem

- Consider relativistic motion

$$\tau_{\gamma\gamma} = \frac{f_p}{\gamma^{2\alpha}} \frac{\sigma_T F D^2}{R_i^2 m_e c^2}$$

$$\tau_{\gamma\gamma} = \frac{10^{13}}{\gamma^{(4+2\alpha)}} f_p \left(\frac{F}{10^{-7}\, erg / cm^2} \right) \left(\frac{D}{3000\, Mpc} \right)^2 \left(\frac{\delta T}{10 mS} \right)^{-2}$$

- If source moves toward the observer with a relativistic velocity →compactness problem can be solved
- $\gamma > 10^{13/(4+2\alpha)}$ 10^2

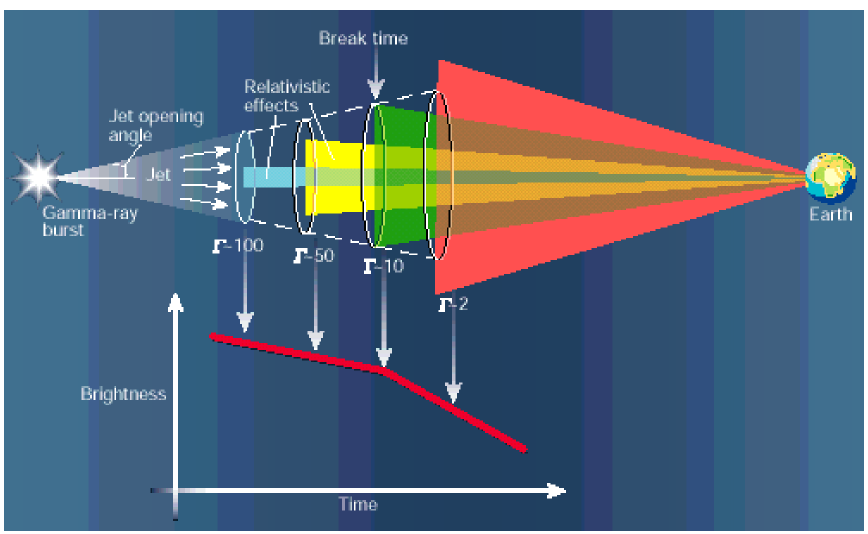
GRB is a Relativistic Jet?

- Some GRB afterglows show achromatic break

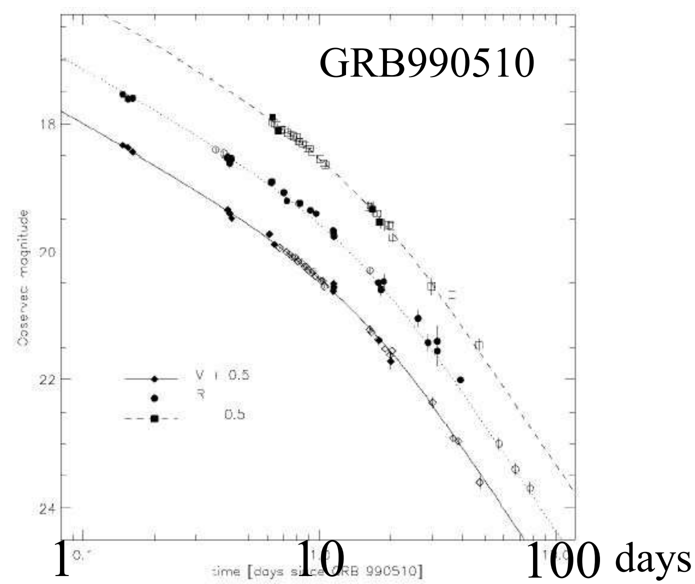
→ It indicates GRB is collimated outflow

- – Jet angle   a few degrees
- – Total energy   narrowly clustered around 10^{51}erg   Frail et al. 1999

→ If supernova-like energy concentrates to jet-like structure, it is possible to make GRB



Schematic picture of achromatic break



Supernova-GRBs Connection

Some evidence is found for a connection between GRBs long burst and supernovae

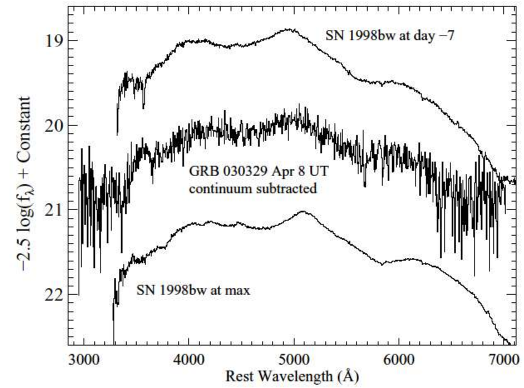
1 Direct evidence

- GRB980425-SN1998bw
  - 10^{48} erg 10^{5} times lower than that of regular GRBs
  - z=0.0085 100 closer than any other GRB
- GRB030329-SN2003dh
  - z=0.169 3rd nearest
  - 10^{50}erg (still lower than that of regular GRBs)

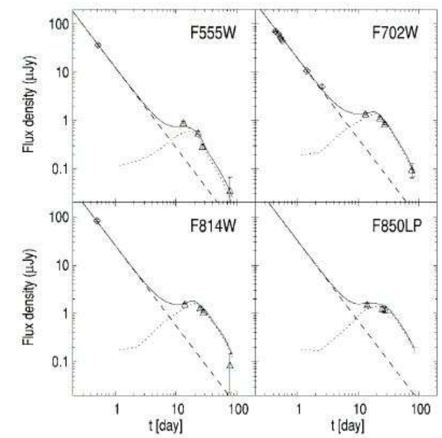
Less certain: GRB031203-SN2003lw (z=0.1; E_{iso} ~$3*10^{49}$ erg)

2 Indirect evidence

- bump in optical afterglow supernova component?
- metal line emission in x-ray afterglow (supernova ejecta?)
- The correlation with Star-forming region



Spectrum of GRB030329



Bump in optical Afterglow (GRB011211)

We think some GRBs are produced by Supernova

Collapsar Model

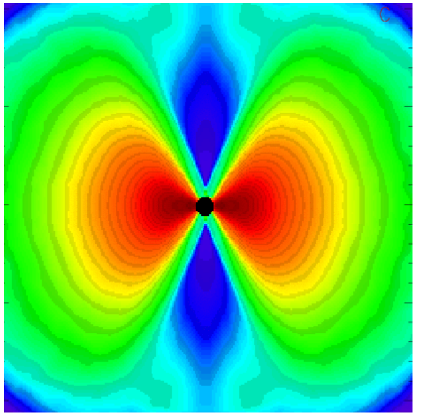
One of the most attractive GRB central engine models, based on the supernova

- Collapsar   rotating massive star (Woosley 1993; MacFadyen & Woosley 1999)
  - Collapse of the iron core of a rotating massive star

  → black hole + disk (or torus)
  - No outward-moving shock (failed SN)
  - Formation of relativistic jet by neutrino-annihilation or MHD process

HD Simulations of a Collapsar

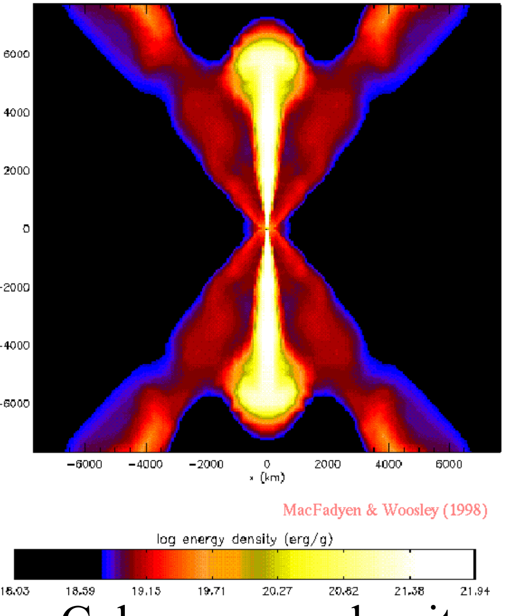
MacFadyen & Woosley 1999; MacFadyen, Woosley, & Heger 2001



Color: density

- 2D hydrodynamic simulations of collapsar
  - 15 M_{sun} presupernova star
  - Realistic Equation of State (EOS) (Neutrino cooling and heating, photodisintegration)
  - Rotation
  - Self gravity
- Formation of jet-like explosion by neutrino annihilation (γ>10)
- They may not fully address the outflow formation mechanism (calculate the energy deposition rate from neutrino annihilation and input this energy from inner boundary)

→ We perform the simulation of jet formation by the MHD process



Color: energy density

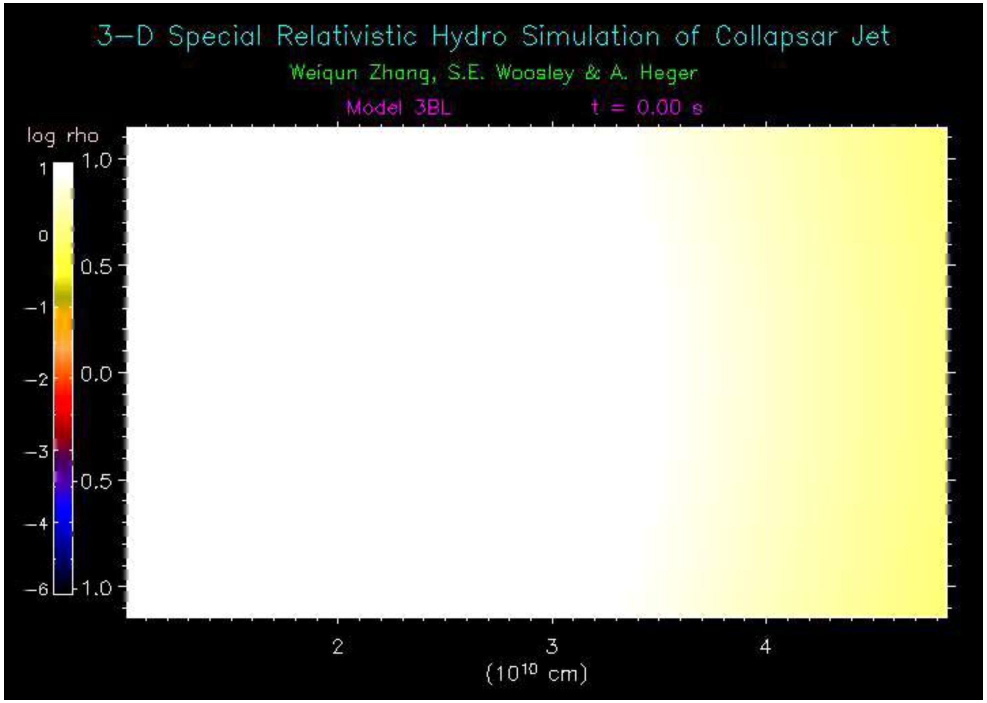
Propagation of Collapsar Jet

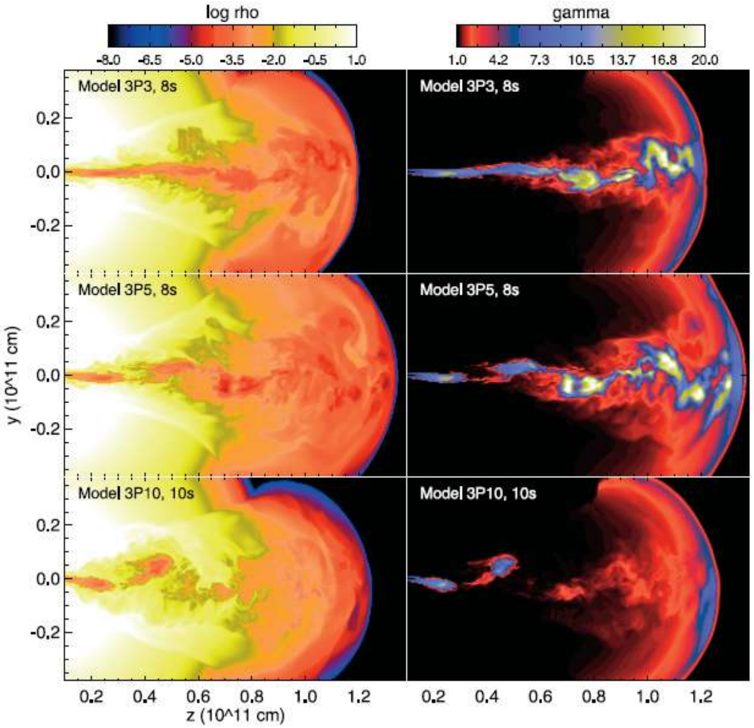
(Zhang, Woosley, & Heger (2004))

• 3D relativistic hydro simulations of relativistic jet propagation into massive stars

• highly relativistic jet (γ~100) → GRB

• moderately relativistic jet (γ~15) →larger polar angle (10°) → X-ray Flash

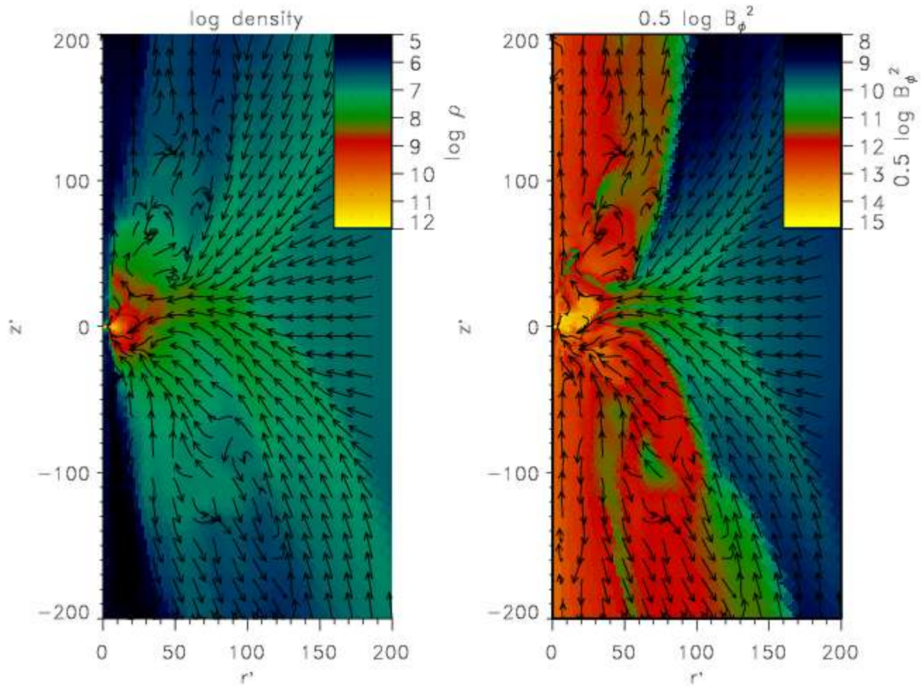




• If the jet changes angle more than 3°in several seconds, it will dissipate, producing a broad beam with inadequate Lorentz factor for GRBs, leads to a X-ray Flash?

MHD Simulations of Collapsar (Proga et al.(2003))

- 2D pseudo-Newtonian MHD simulations of collapsar.
  - Pseudo-Newtonian MHD (Based on ZEUS)
  - $25M_{sun}$ presupernova star (Woosley & Weaver 1995)
  - Rotation and weak radial B field
  - Realistic EOS (Neutrino cooling, photodisintegration of helium)
  - resistive heating



- Strong polar outflow are able to be launched, accelerated by MHD effects.
- Outflow is Poynting flux-dominated.

Relation between GRBs and Magnetic Field

- There are several motivations for considering strong magnetic fields
  - Electromagnetic energy is clean
  - GRB central engine models invoke a rapid rotating BH + disk system
  - The magnetic field is amplified via dynamo effect quickly
  - Magnetic field is a possible tool to extract energy from the engine
  - Magnetic field is helpful to collimate the jet
- Observation (although controversial)
  - Strong gamma-ray polarization : RHESSI 80%±20% prompt emission $\rightarrow$ strongly magnetized central engine

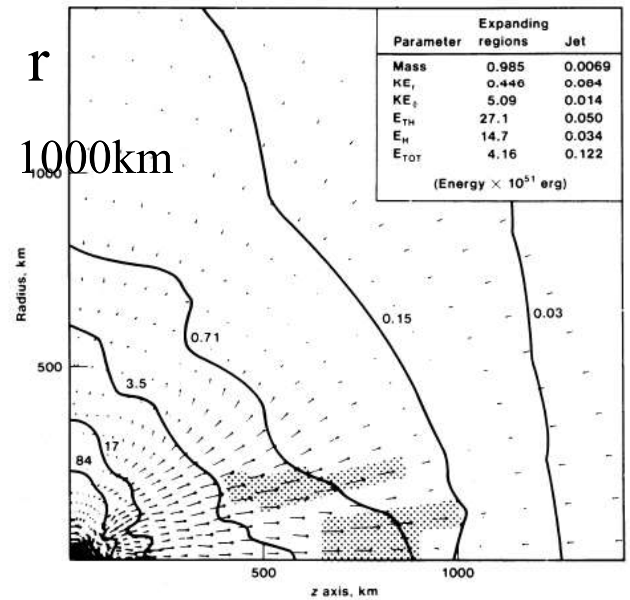
Simulation of Gravitational Collapse with Rotation and Magnetic Field

Leblanc & Wilson (1970), Symbalisty (1984)

- 2.5D non-relativisitc MHD + neutrino transport
- Initial conditions
  - Inner core $2M_{sun}$,Total $15M_{sun}$
  - Rigid-like rotation and dipole-like magnetic field
  - E_{mag}/E_{gr}=0.56%; E_{rot}/E_{gr}=4.5%
- Results
  - The formation of quasi-static core core density $4.4*10^{14}g/cm^3$
  - The formation of jet by the effect of rotation and magnetic field
    - Maximum velocity: 4.4×10^8 cm/s
    - Total energy: 1.22×10^{50} erg
    - Magnetic field: 5×10^{14}G

r

1000km



Z (rotation axis)

- Although this simulation is not applied to collapsar model, it may be possible to obtain the same result from the simulation of collapsar model

Purpose of Present Study

- We consider the collapsar model with magnetic field as a central engine of GRB
- Focus on the generation of a relativistic jet by the effect of magnetic field and general relativity

↓

Can it produce the relativistic outflow based on GRBs?

We simulate it by using the general relativistic MHD code (Koide et al. 2000)

4D General Relativistic MHD Equation

- General relativistic equation of conservation laws and Maxwell equations:

$$\nabla_{\cdot} (n U^{\nu}) = 0$$ (conservation law of particle-number)

$$\nabla T^{\mu\nu} = 0$$ (conservation law of energy momentum)

(Maxwell equations)

$$\partial F_{\nu\lambda} + \partial_{\nu} F_{\lambda\mu} + \partial_{\lambda} F_{\mu\nu} = 0$$

$$\nabla F^{\mu\nu} = - J^{\nu}$$

- Frozen-in condition: $F_{\nu\mu} U^{\nu} = 0$

- metric $ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu}$;

$g_{00} = - h_0^2$; $g_{ii} = - h_i^2$;

$g_{0i} = - h_i^2 \omega_i \; (i=1,2,3)$; $g_{ij} = 0 \; (i \neq j)$

We neglect the evolution of metric and the essential micro physics (we use gamma-law EOS)

n: proper particle number density. p : proper pressure. c: speed of light.

e : proper total energy density, $e = mnc^2 + p / (\Gamma - 1)$.; $\Gamma = 5/3$

m : rest mass of particles. Γ: specific heat ratio.

$U^{\mu\upsilon}$: velocity four vector. $A^{\mu\upsilon}$: potential four vector. $J^{\mu\upsilon}$: current density four vector.

$\nabla^{\mu\nu}$: covariant derivative. $g_{\mu\nu}$: metric.

$T^{\mu\nu}$: energy momentum tensor, $T^{\mu\nu} = p g^{\mu\nu} + (e+p) U^{\mu} U^{\nu} + F^{\mu\sigma} F^{\nu}{}_{\sigma} - g_{\mu\nu} F^{\lambda\kappa} F_{\lambda\kappa}/4$.

$F_{\mu\nu}$: field-strength tensor, $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$.

Vector Form of General Relativistic MHD Equation
(3+1 Formalism)

$$\frac{\partial D}{\partial t} = - \quad \cdot [\alpha D(\mathbf{v} + \mathbf{v}_{\mathrm{H}})]$$

general relativistic effect

(conservation law of particle-number)

$$\frac{\partial \mathbf{P}}{\partial t} = - \quad \cdot [\alpha(\mathbf{T} + c\boldsymbol{\beta}\mathbf{P})] - \left(D + \frac{\varepsilon}{c^2} \right) \nabla(c^2\alpha) + \alpha \mathbf{f}_{\mathrm{curv}} - \mathbf{P} : \boldsymbol{\sigma}$$

special relativistic effect

(equation of motion)

$$\frac{\partial \varepsilon}{\partial t} = - \quad \cdot [\alpha(c^2\mathbf{P} - Dc^2\mathbf{v} + \varepsilon \mathbf{v}_{\mathrm{H}})] - (\nabla\alpha) \cdot c^2\mathbf{P} - \mathbf{T} : \boldsymbol{\sigma}$$

(energy equation)

$$\frac{\partial \mathbf{B}}{\partial t} = - \quad \times [\alpha(\mathbf{E} - c\boldsymbol{\beta} \times \mathbf{B})]$$

$$\mathbf{J} + \rho_{\mathrm{e}} c\boldsymbol{\beta} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \left[\alpha \left(\mathbf{B} + \frac{\boldsymbol{\beta}}{c} \times \mathbf{E} \right) \right]$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \rho_{\mathrm{e}} = \frac{1}{c^2} \nabla \cdot \mathbf{E}$$

(Maxwell equations)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

(ideal MHD condition)

D: density
P: momentum density
T: energy-momentum tensor
ε : energy density

Where

$$\alpha = \sqrt{h_0^2 + \sum_{i=1}^{3} \left(\frac{h_i \omega_i}{c} \right)^2}$$: (Lapse function)

$$\beta^i = \frac{h_i \omega_i}{c\alpha}$$: (shift vector)

$$\mathbf{v}_{\mathrm{H}} = c\boldsymbol{\beta}$$: (shift velocity)

$$f_{\mathrm{curv}}^i = \sum_{j=1}^{3} \left(G_{ij} T^{ij} - G_{ji} T^{jj} \right) \qquad G_{ij} = -\frac{1}{h_i h_j} \frac{\partial h_i}{\partial x^j} \qquad \sigma_{ij} = \frac{h_i}{h_j} \frac{\partial \omega_i}{\partial x^j}$$

Vector Form of General Relativistic MHD Equation (3+1 Formalism)

Conserved quantities → primitive variables:
2-variable Newton-Raphson iteration method

$$x \equiv \gamma - 1 \text{ and } y \equiv \gamma(\hat{\mathbf{v}} \cdot \hat{\mathbf{B}})/c^2$$

$$x(x+2)\left[\Gamma R x^2 + (2\Gamma R - d)x + \Gamma R - d + u + \frac{\Gamma}{2}y^2\right]^2$$
$$= (\Gamma x^2 + 2\Gamma x + 1)^2[f^2(x+1)^2 + 2\sigma y + 2\sigma x y + b^2 y^2], \tag{80}$$

$$\left[\Gamma(R - b^2)x^2 + (2\Gamma R - 2\Gamma b^2 - d)x + \Gamma R - d + u - b^2 \right.$$
$$\left. + \frac{\Gamma}{2}y^2\right]y = \sigma(x+1)(\Gamma x^2 + 2\Gamma x + 1), \tag{81}$$

$R = D + \epsilon/c^2$, $d = (\Gamma - 1)D$, $u = (1 - \Gamma/2)\hat{B}^2/c^2$, $\jmath = \hat{P}/c$, $b = \hat{B}/c$, and $\sigma = \hat{\mathbf{B}} \cdot \hat{\mathbf{P}}/c^2$

$$\gamma = 1 + x,$$

$$p = \frac{(\Gamma - 1)[\epsilon - xDc^2 - (2 - 1/\gamma^2)B^2/2 + (cy/\gamma)^2/2]}{[\Gamma x(x+2) + 1]}$$

$$\hat{\mathbf{v}} = \frac{\mathbf{P} + (y/\gamma)\mathbf{B}}{D + \{\epsilon + p + B^2/2\gamma^2 + (cy/\gamma)^2/2\}/c^2}.$$

Metric

Metric of Kerr space-time (Boyer-Lindquist coordinates: (R, ϕ , θ))

$$h_0 = \sqrt{1 - \frac{2r_g R}{\Sigma}}, \quad h_1 = \sqrt{\frac{\Sigma}{\Delta}}, \quad h_2 = \sqrt{\Sigma}, \quad h_3 = \sqrt{\frac{A}{\Sigma}} \sin\theta,$$

$$\omega_1 = \omega_2 = 0, \quad \omega_3 = \frac{2cr_g^2 aR}{A}$$

Where

$$\Sigma = R^2 + (ar_g)^2 \cos^2\theta \qquad \Delta = R^2 - 2r_g R + (ar_g)^2 \cos^2\theta,$$

$$A = \{R^2 + (ar_g)^2\}^2 - \Delta(ar_g)^2 \sin^2\theta$$

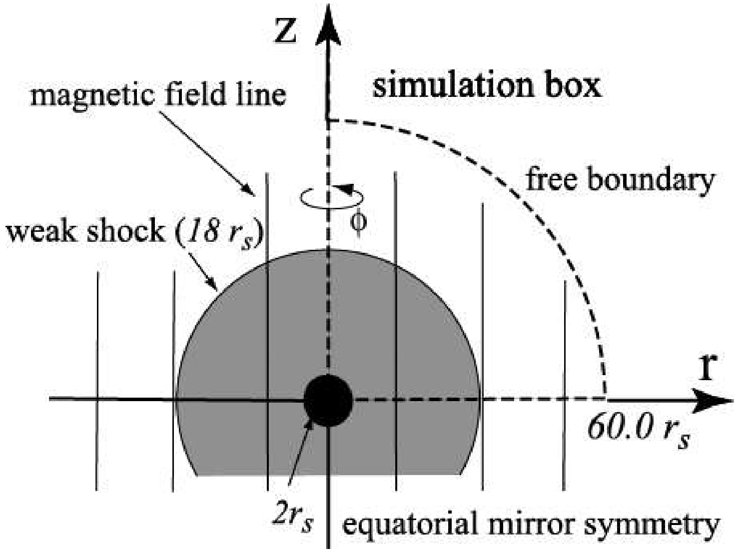
r_g=GM/c^2: gravitational radius
a=J/J_{max}: rotation parameter
J: angular momentum

When a=0.0, metric → the non-rotating black hole (Schwarzschild space-time)

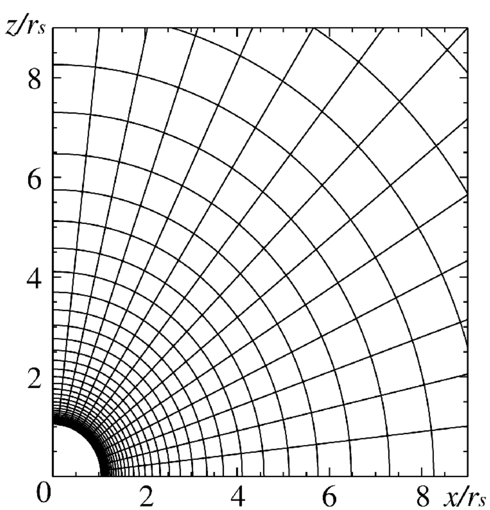
Simulation Model

- We assume the following initial conditions
  - Iron core of massive star collapse
  - Stellar mass black hole is formed
  - Stellar matter fall toward the central BH
- Simulation Code
  - 2.5D General relativistic MHD code Koide et al. 1999, 2000
- Initial Conditions
  - A black hole non-rotating or rotating is at the origin
  - We employ the profiles of the density, pressure and radial velocity as a guide for the scale free structure from the results of 1D supernova simulations Bruenn, 1992; 20 M_{sun} model
  - Rotation profile a function of the distance from the rotation axis
  - Initial magnetic field uniform and parallel field Wald solution
- Numerical scheme
  - Simplified TVD scheme Davis 1984
- Simulation region 1.4(a=0.999), $2r_s$(a=0.0) R $60r_s$, 0 /2
- Mesh number 120×120

Simulation Model

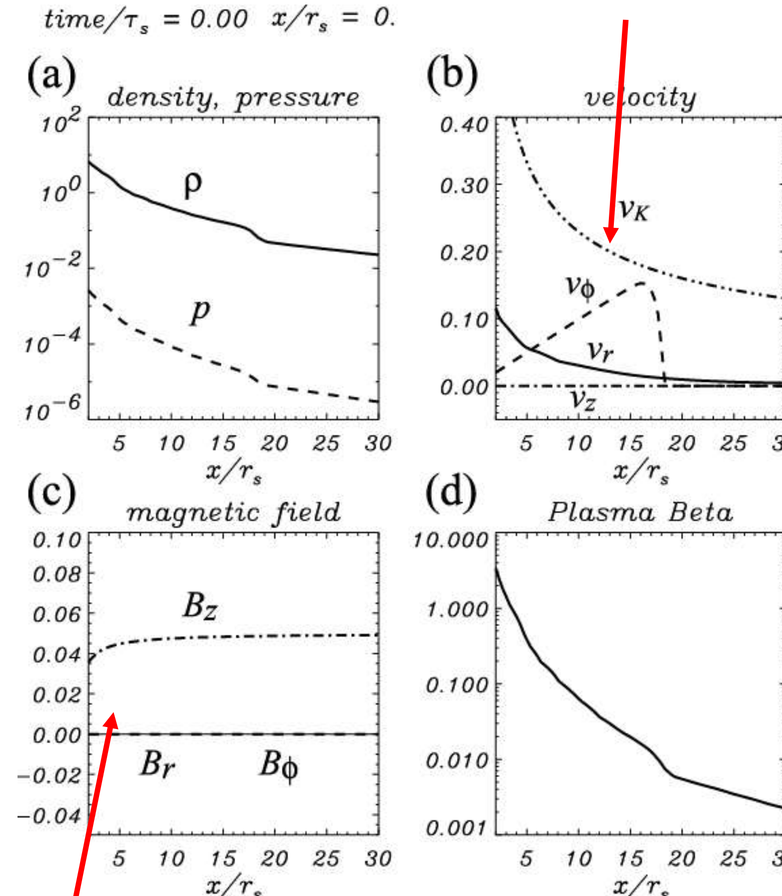


Schematic picture of our simulation
Gray: rotation



Distribution of mesh point

Rigid-like rotation



Uniform magnetic field

The distribution on equatorial plane

Bruenn, 1992

Rotating Black Hole - two cases

- Co-rotating case $a=0.999$
  - The rotation of black hole is same direction with respect to the rotation of stellar matter
- Counter-rotating case ($a=-0.999$)
  - The rotation of black hole is opposite direction with respect to the rotation of stellar matter

  → Although this is unrealistic in the collapsar model, we performed it as a numerical experiment

Kerr BH
Co-rotating case $a=0.999$

Unit

Length: r_s

(Schwarzschild radius)

time: $\tau_s = r_s/c$

(when $M_{BH} \sim 3M_{sun}$

$1r_s \sim 10^6$cm; $1\tau_s \sim 0.03$ms,

when $\rho \sim 10^{10}$ g/cm^3

$B_0 \sim 10^{14}$G)

Parameter

B_0:0.05, V_0:0.01

E_{mag}:1.68×10^{-3}

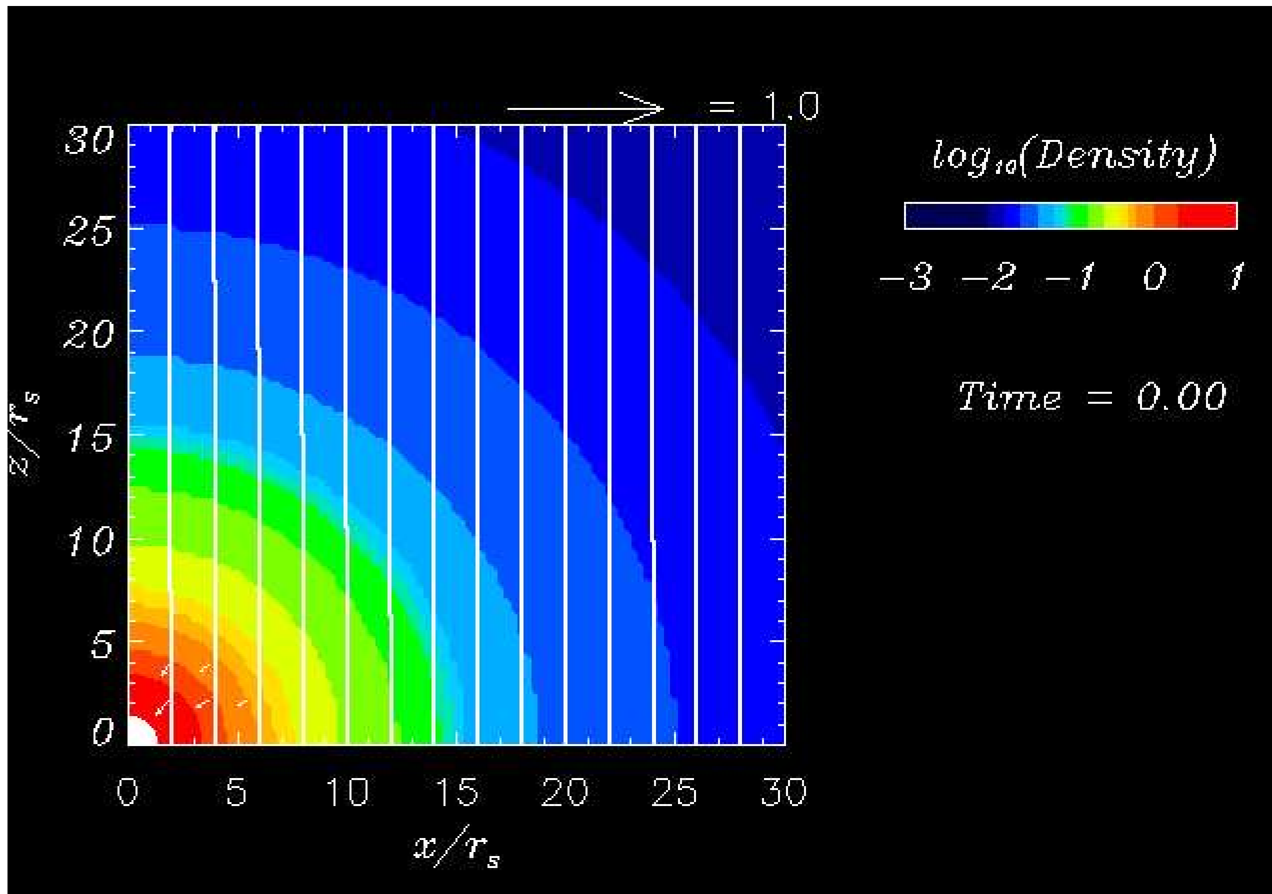
E_{rot}:5.36×10^{-2}

$E_{mag} = V_{A0}{}^2 / V_{K0}{}^2$

$E_{ro} = V_{\phi}{}^2 / V_{K0}{}^2$

Subscript "0" is the value at $r=3r_s$

color density, line: magnetic field line, vector poloidal velocity



Kerr BH
Counter-rotating case (*a=-0.999*)

Unit

Length: r_s

(Schwarzschild radius)

time: r_s/c

(when M_{BH}~$3M_{sun}$

$1r_s$~10^6cm; $1\tau_s$~0.03ms,

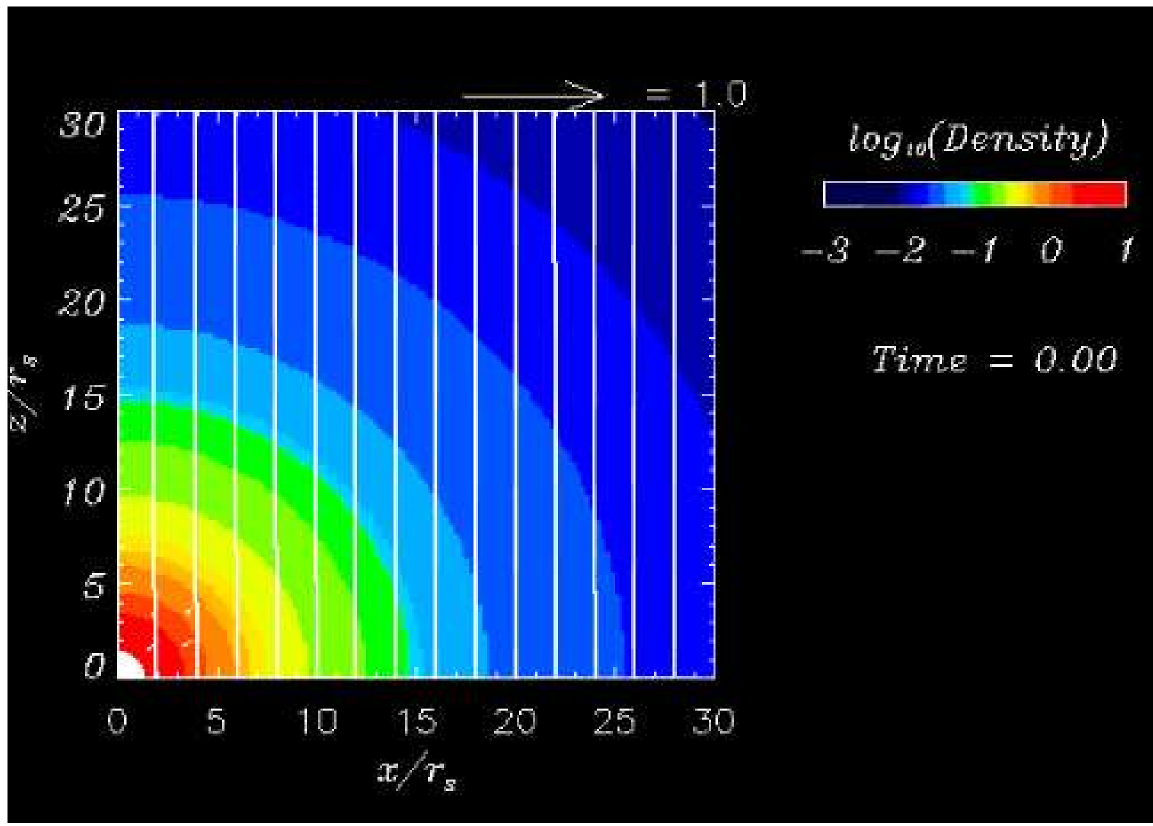
when r~10^{10} g/cm^3

B_0~10^{14}G)

Parameter

B_0:0.05,V_0:0.01

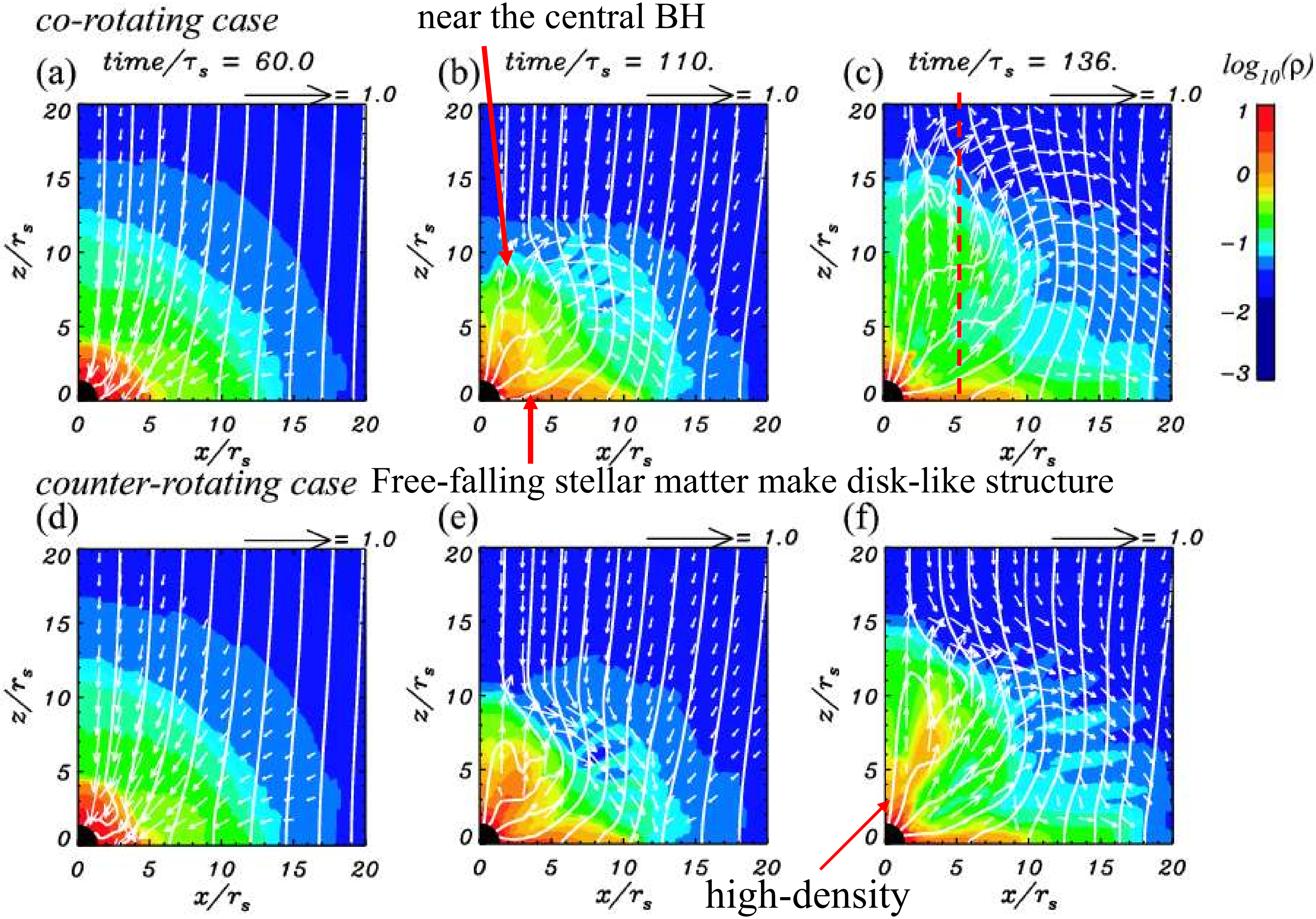
color density, line: magnetic field line, vector poloidal velocity



Snapshot of density

color density
line magnetic field lines

Jet-like outflow is ejected
near the central BH



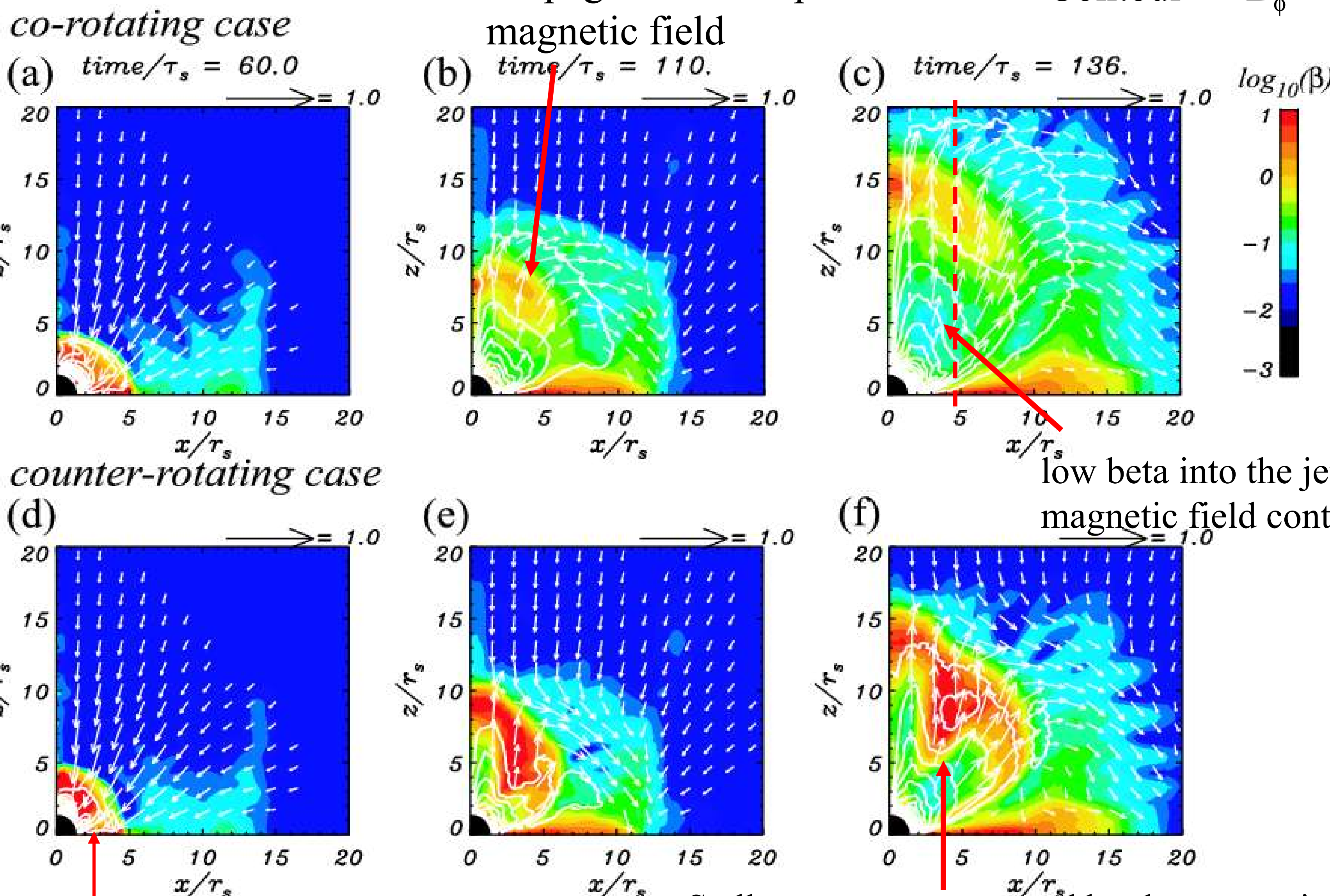
Free-falling stellar matter make disk-like structure

high-density

Snapshot of Plasma Beta

Color: $\beta = P_{gas}/P_{mag}$

Contour B_ϕ



Propagation of amplified magnetic field

low beta into the jet
magnetic field contribute

amplified magnetic field

Stellar matter compressed by the magnetic field
→ High beta structure

Co-rotating case
a=0.999

$x/r_S=5,\ t/\tau_S=136$ ($1\tau_s \sim 0.03$ms)

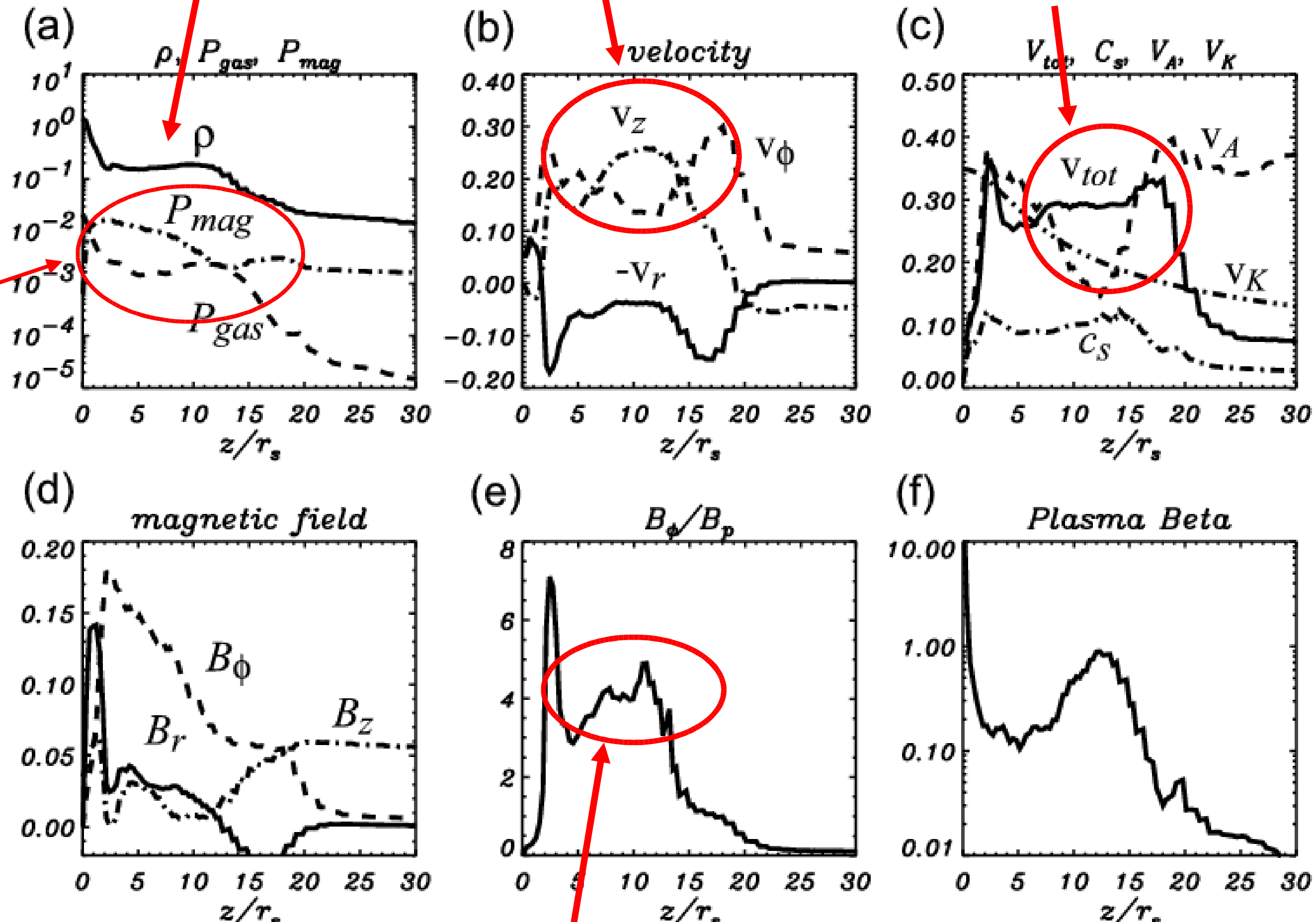
Properties of Jet

High density into the jet

V_z is dominant component

Jet velocity exceeds the Kepler velocity
Jet is mildly relativistic (~*0.3c*)

Magnetic pressure is dominant



Jet is strongly twisted
= pinching force operates
the collimation of jet

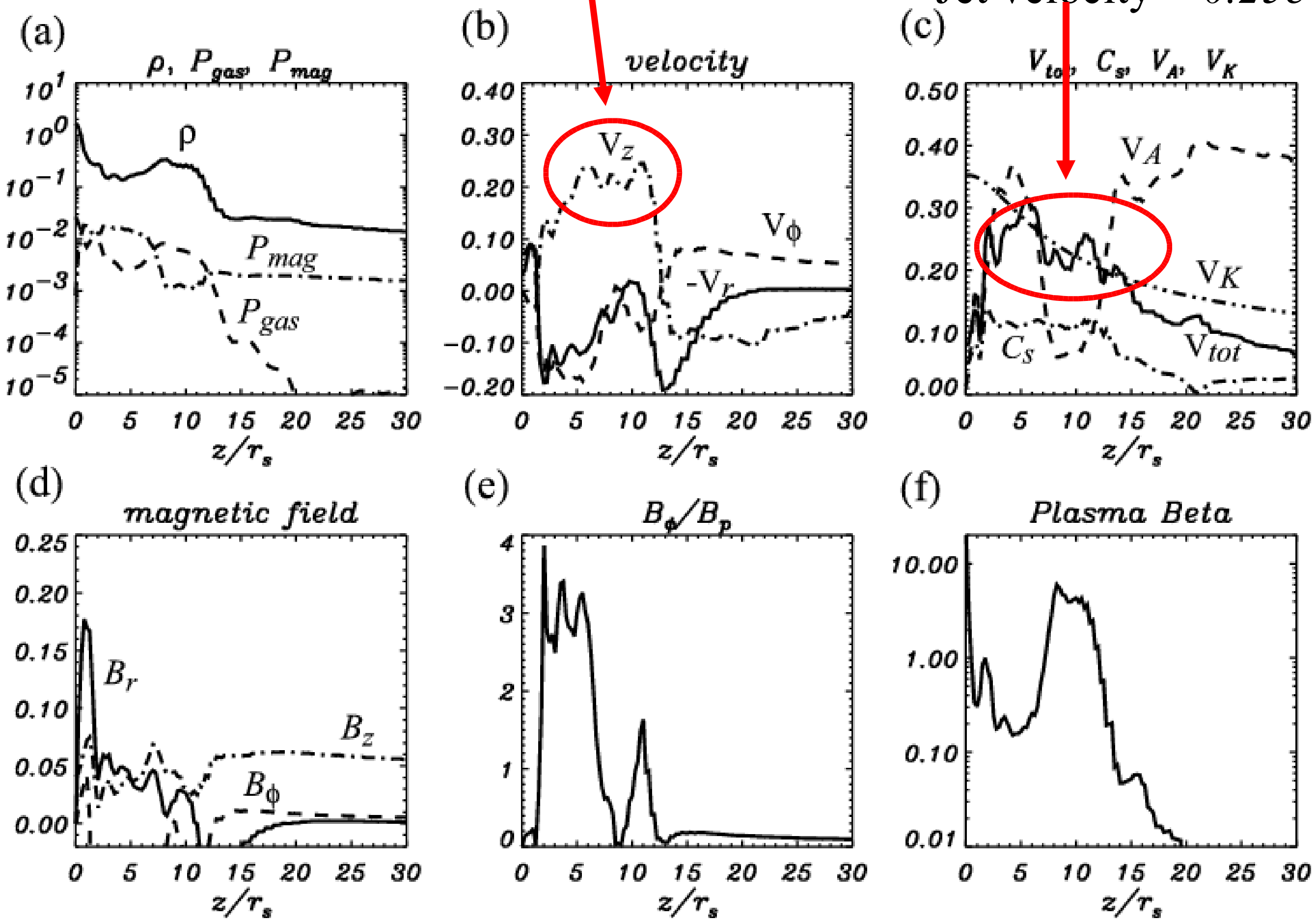
Counter-rotating case
a= -0.999

Properties of Jet

$x/r_S=5,\ t/\tau_S=136$ ($1\tau_s$~0.03ms)

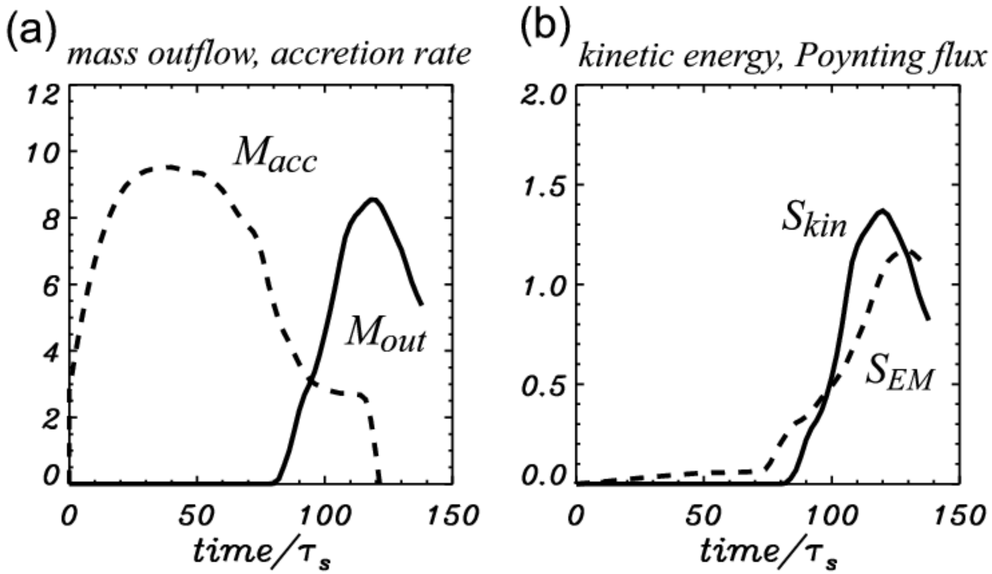
V_z is dominant component

Jet velocity is comparable to the Kepler velocity
Jet velocity ~ *0.25c*



Comparison of time evolution of each flux

co-rotating case At z/r_s~15



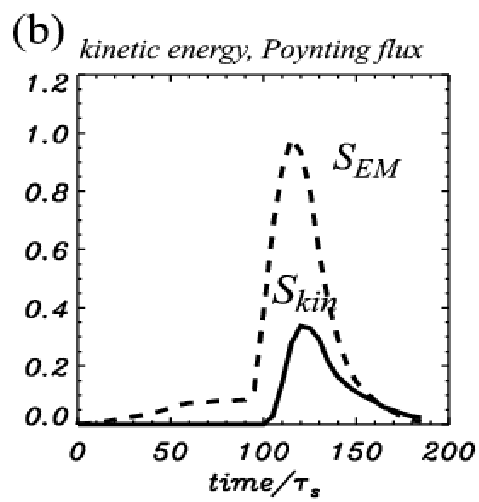
Kinetic energy flux is comparable to Poynting flux

- E_{kin} of jet depends on the scaling of density
  - We assume ρ 10^{10} g/cm^3
- Estimates E_{kin} of Jet

→ E_{jet} 10^{51}erg

This is comparable with the standard energy of GRBs 10^{51}erg

Non-rotating case



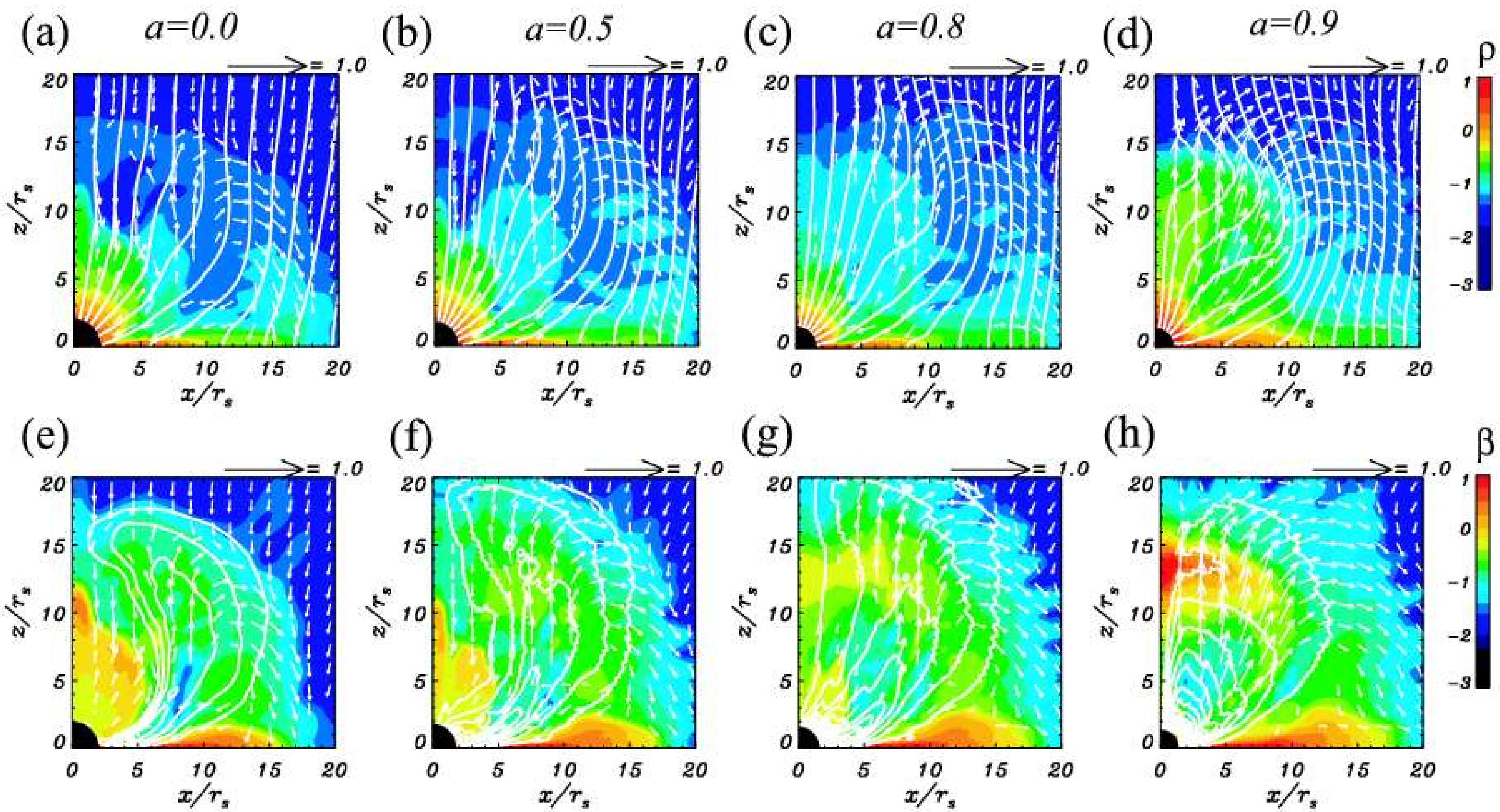
Dependence on BH rotation

t/τ =136,1τ_s~0.03ms a: Black hole rotation parameter (a=J/J_{max})

Co-rotating case   ***slow***   ←   **BH rotation**   →   ***fast***

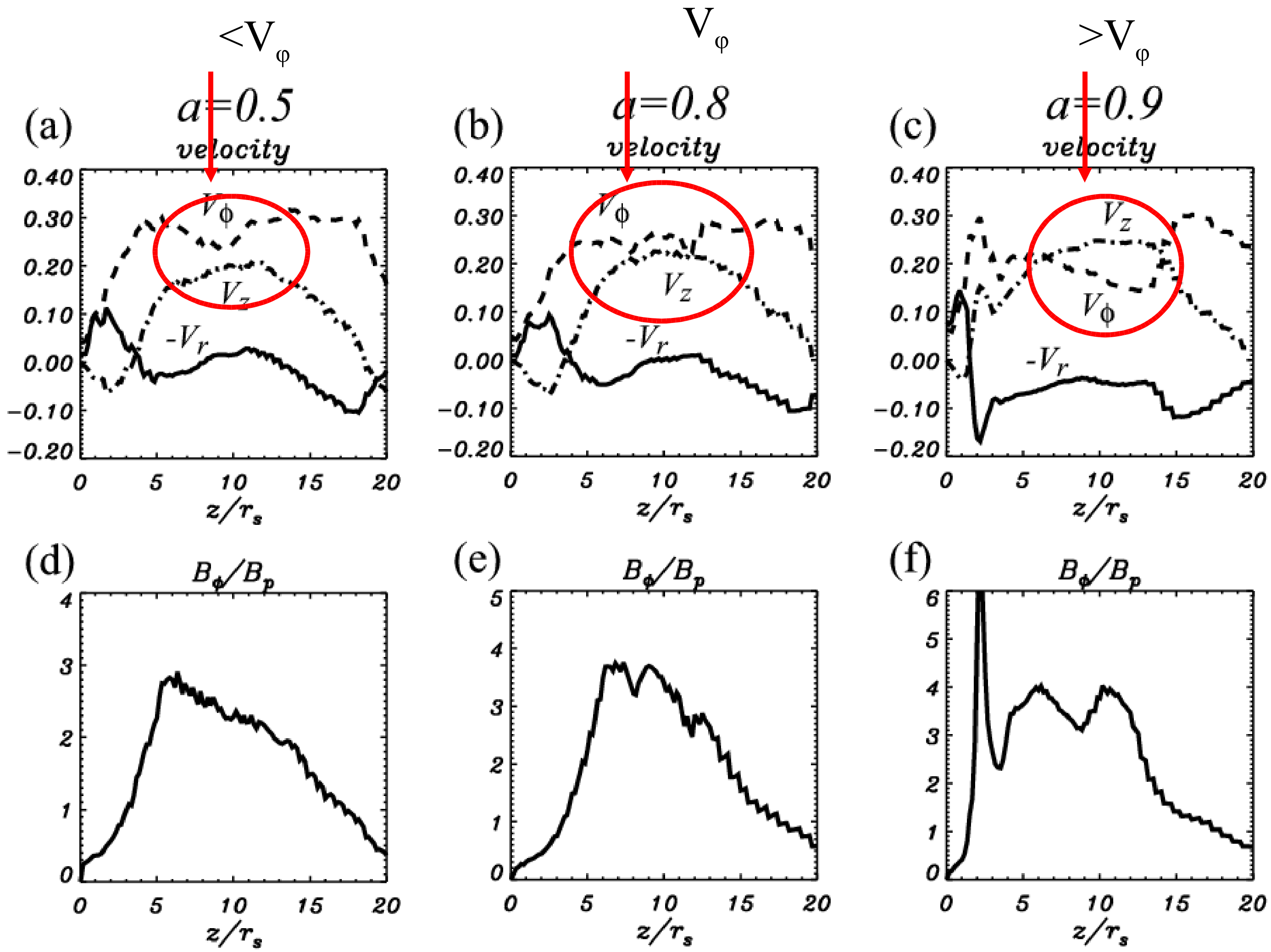
Color density
Line: magnetic field lines

Color Plasma beta
contour B_φ



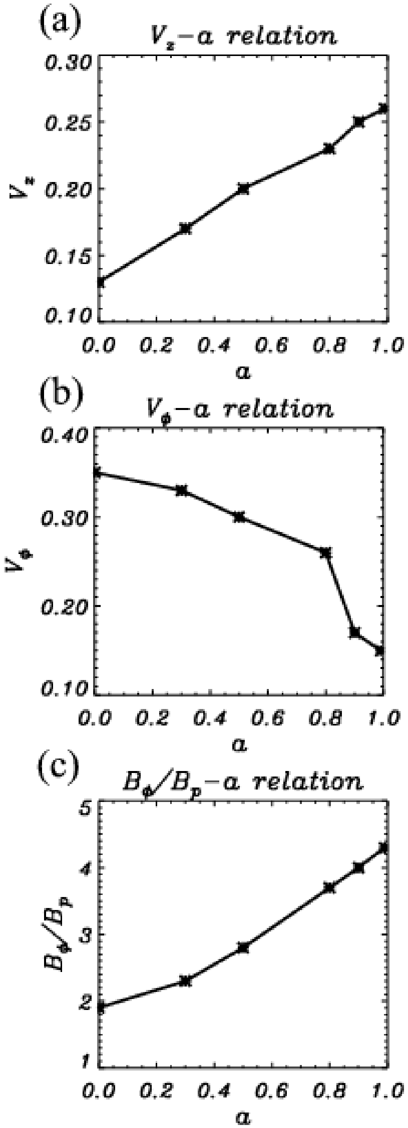
- For smaller values of the rotation parameter,
  - the jet is ejected from *more* outer regions
  - the propagation of the amplified magnetic field as Alfven waves is slower and is seen more clearly

Dependence on the Rotation parameter



When the rotation of black hole is faster, magnetic twist becomes larger

Dependence on the Rotation parameter



- When the rotation parameter of BH↑→ V_p of jet and magnetic twist↑, V_ϕ of jet ↓
- These results are based on how much the frame-dragging effect works on the twisting of magnetic field
- the rotation of BH ↑ → the magnetic field is twisted strongly by the frame-dragging effect → The stored E_{mag} by the twisted magnetic field is converted to E_{kin} of jet directly rather than propagating as Alfven waves → poloidal velocity of jet ↑

Physical Reason

Time evolution of toroidal magnetic field in Newtonian case

$$\frac{\partial B_\phi}{\partial t} = \omega B_p, \qquad \omega \quad \text{angular velocity}$$

Angular velocity consists of the rotation of matter and frame (space-time)
If the magnetic twist occurs far from black hole

$$\omega \propto a.$$

From this

$$\frac{B_\phi}{B_p} \sim \omega t \propto a.$$

The magnetic twist becomes faster proportional to the rotation of black hole

Physical Reason

The upward motion of the fluid is induced by J×B force
The equation of motion in z-direction

$$\rho \frac{\partial v_z}{\partial t} \sim \nabla \left(\frac{B_\phi^2}{4\pi} \right) \sim \frac{1}{z} \left(\frac{B_\phi^2}{4\pi} \right),$$

Which can be rewritten as

$$v_z \sim \frac{1}{\rho} \frac{t}{z} \left(\frac{B_\phi^2}{4\pi} \right)$$

B_φ/B_p>1→time scale is determined by the propagation time scale in the toroidal direction of Alfven wave
Thus z/t $V_{A\varphi}$

$$v_z \sim \frac{1}{\rho} \frac{1}{v_{A\phi}} \left(\frac{B_\phi^2}{4\pi} \right) \propto B_\phi \propto a.$$

Poloidal velocity of jet becomes faster proportional to the rotation of BH

Physical Reason

On the other hand, the equation of motion in the toroidal direction

$$\rho \frac{\partial v_\phi}{\partial t} \sim \nabla \left(\frac{B_\phi B_z}{4\pi} \right) \sim \frac{1}{z} \left(\frac{B_\phi B_z}{4\pi} \right)$$

Using z/t V$_{a\varphi}$

$$v_\phi \sim \frac{1}{\rho} \frac{1}{v_{A\phi}} \left(\frac{B_\phi B_z}{4\pi} \right) \propto B_z = const.$$

This approximately explains the dependence of v_ϕ for a 0.8

However, the exact solution depends on the region where the jet is ejected

Discussion　Applied to the GRB Jet

- Jet velocity　mildly relativistic 0.3c
- Too slow for the GRB jets→ have to consider other acceleration mechanisms
  - Steady solution (Begelman & Li 1994; Daigne & Drenkhahn 2002)
    - The magnetic field lines diverge with radius more quickly than in the monopole field ($B_p \propto r^{-a}$; $a>2$)

    → The outflow is highly-accelerated
    - This solution is not self-consistent　the geometry of the magnetic field is not solved
    - May not maintain the collimated structure
  - Dissipation-induced flow acceleration mechanism (Spruit, Daigne & Drenkhahn 2001; Sikora et al. 2003)
    - Energy transport as Poynting flux and releases by reconnection

    →　converts to directly into radiation and kinetic energy of jets

Discussion (cont.) Application to other models

- On the other hand, our results can be applied to baryon-rich outflows associated with failed GRBs
  - The jet velocity is so slow that it cannot produce the GRBs → It is a fireball with a high baryonic load
  - example SN 2002ap
    - Although it is not associated with a GRB, it has a jet
      Kawabata et al. 2002; Totani 2003
    - Jet velocity 0.23c, E_{kin} of jet 5×10^{50}erg
    - It can be explained by our simulations

Summary and Conclusions

- The formation of disk-like structures and generation of jet-like outflow from collapsar model are reproduced
- The magnetic field is twisted by the rotation of stellar matter and the frame-dragging effect and propagates outward as an Alfven wave
- Jet-like outflows are formed and accelerated by the effect of magnetic field, and they are mildly relativistic v 0.3c)
- In the co-rotating case, the kinetic energy flux is comparable to the Poynting flux

Summary and Conclusions (cont.)

- As the rotation of the BH increases, the poloidal velocity of the jet and magnetic twist increases gradually and toroidal velocity of the jet decreases. Because the magnetic field is twisted strongly by the frame dragging effect, it can store much magnetic energy and converts to kinetic energy of the jet directly
- Although the jets in our simulations are imperfect as a model for GRBs, they can explain the baryon-rich outflow associated with failed-GRBs